

## Task Design and Implementation as a Two-way Activity: The Case of Preschool Teachers

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### Abstract

This paper demonstrates how professional development which focuses on task design principles can impact on both preschool teachers' knowledge and practice. Specific design principles for preschool mathematics tasks are presented and exemplified. Teachers chose several tasks to implement with individual children in their preschool classes, video-taped the task implementations, and discussed the experience during program sessions. Program sessions were recorded and analyzed qualitatively. Findings show that the participating preschool teachers become attentive to their students' knowledge as well as to task features. This attentiveness allowed them to modify task to target specific knowledge constructs.

**Key words:** preschool teachers, professional development, task design principles, counting and enumerating.

## Tvoření úloh a jejich implementace jako obousměrný proces: Příklad učitelů mateřských škol

### Abstrakt

Článek ukazuje, jaký vliv může mít další vzdělávání učitelů zaměřené na tvorbu úloh na jejich znalosti a praxi. Jsou představeny konkrétní principy tvoření úloh pro mateřské školy a tyto principy jsou ilustrovány na příkladech. Učitelé si vybírali několik úloh, které použili s jednotlivými dětmi ve svých třídách mateřských škol; implementace úloh byla nahrávána na video a analyzována při společných setkáních. Také setkání byla nahrávána na video a proběhla jejich kvalitativní analýza. Výsledky ukazují, že učitelé v mateřských školách pozorněji sledovali znalosti svých žáků i charakteristiky úloh. To jim umožnilo upravit úlohy tak, aby směřovaly ke konkrétním matematickým konceptům.

**Klíčová slova:** učitelé mateřských škol, další vzdělávání, zásady pro tvorbu úloh, počítání a vyčíslování.

There are several ways in which teachers can engage with mathematical tasks during professional development. Many studies suggest having teachers analyse existing tasks as opposed to designing their own tasks. These tasks can be textbook tasks (Nicol & Crespo, 2006; Watson & Sullivan, 2008) or tasks specifically designed by a designer or researcher (Swan, 2007). Perhaps because tasks are often written by designers and not by teachers, few professional development programs focus specifically on design principles and design features of tasks. Yet, there are instances when teachers have no choice but to design mathematical tasks for their own classrooms. Such is the case with preschool teachers in Israel where a mandatory mathematics preschool curriculum exists, but few curricular materials are available. When materials and tasks are available, they are not usually accompanied by teacher guides, which do accompany primary and secondary school textbooks in Israel. For these preschool teachers, it is especially important to appreciate the design process so that they may design mathematical tasks on their own or use chosen tasks wisely. For us, when working with preschool teachers, it is important to understand not only how an intervention might affect preschool teachers' practice related to task design but also how practice with tasks may inform further decisions related to tasks and instruction.

This paper addresses the general question of what may inform a teacher's decision to change a specific feature of some task. It focuses on two possibilities: the teacher's sensitivity to his or her students' knowledge of mathematics and the teacher's experiences of implementing the task. However, we also believe that a prerequisite for wisely changing a task feature is that the teacher be informed of the task's features and design as well as the principles that lay behind the design. Furthermore, Watson and Sullivan (2008) claimed that critical analysis of tasks may stimulate teachers' theorising about students' learning. Thus, it is also possible that being knowledgeable of a task's design may increase what a teacher may learn about students from implementing the task. We propose the possibility of a chain relationship: knowing the design principles behind a task and being aware of the specific features of a task may impact on what a teacher may learn from implementing that task which in turn may impact on how the teacher changes specific features of that task. In this paper we explore this possibility as we describe a professional development program for preschool teachers which emphasized task design and report on some of the results of this program. In the process, we also ask ourselves how design considerations can facilitate teachers' adaptation of tasks.

Henningsten and Stein (1997) defined a mathematical task as "a classroom activity, the purpose of which is to focus students' attention on a particular mathematical concept, idea, or skill" (p. 528). We adopt this definition for our study. In line with Watson and Mason (2006), we view a "collection of procedural questions or tasks . . . as a single object, with individual questions seen as elements in a mathematically and pedagogically structured set" (p. 91). In preschool, a series of procedural tasks may include counting the number of objects in a basket, then counting the same objects placed in a row, and then counting the same number of objects arranged in a circle. Also in line with Watson and Mason (2006) we apply variation theory to discuss not only how learning experiences may vary but also how variation may be used as a tool for constructing different tasks that are conceptually related. In the above example, all tasks are related to the concept of enumeration. The arrangement of items to be counted is varied.

This paper begins by briefly describing a professional development program for practicing preschool teachers, the role mathematical tasks played in this program,

and how task design became explicit to the teachers. It then reviews results of teachers' implementation of mathematical tasks focusing on teachers' analysis of their students' conceptions and how implementation of the tasks enhanced teachers' sensitivity to task design. Examples were chosen for their ability to highlight different aspects of task design. Finally, implications and questions are discussed.

## 1 THE PROFESSIONAL DEVELOPMENT PROGRAM

For several years, we have been providing professional development for preschool teachers guided by the Cognitive Affective Mathematics Teacher Education (CAMTE) framework (e.g. Tirosh, Tsamir, Levenson, Tabach & Barkai, 2011). Our aims are to promote teachers' knowledge and self-efficacy for teaching mathematics to young children. Like many other programs, we engage teachers with mathematical tasks. To begin with, we simulate playing with children with these tasks in order to promote teachers' knowledge of mathematical concepts such as equivalence and geometrical shapes. This type of simulation allows teachers to confront, in a gentle and respectful manner, their own conceptions and serves as a springboard for a thorough discussion of common errors, promoting also their knowledge of children's conceptions. In time, teachers are invited to take part in these simulations, promoting their knowledge of tasks and teaching. Many of the tasks we bring to our program are designed by us and have been implemented in preschools, by us, as well as by preschool teachers who have participated in our programs. This section begins by laying out the principles that guided our design of preschool mathematical tasks. It then describes how the design of these tasks became explicit to teachers.

Several principles guide our design of mathematical tasks to be used with preschool children. First, the mathematical concepts and competencies embedded in the tasks stem from the mandatory Israel National Mathematics Preschool Curriculum (INMPC, 2008). Second, as mentioned above, series of tasks may be conceived as a single object but are varied in such a way as to highlight separate competencies. For example, when working with preschool teachers and their young students, we differentiate between counting and enumerating skills. Counting refers to saying the number words in the proper order and knowing the principles and patterns in the number system as coded in one's natural language (Baroody, 1987). Enumerating refers to counting objects for the purpose of saying how many. On a day-to-day basis, the term enumerating is hardly used, but this professional term is used in the INMPC and helps preschool teachers to distinguish between the different skills we wish to promote among children. Gelman and Gallistel (1978) outlined five principles of counting objects: the one-to-one principle, the stable-order principle, the cardinal principle, the abstraction principle, and the order-irrelevance principle. While a task may promote more than one competency at a time, a key principle of our design is that it should be clear which of the competencies is being targeted at each point of the task. In light of the above design principle, careful attention is paid to the exact wording of the instructions and questions involved in a task. A third design principle is that the objects used in the task be readily available to the preschool teacher and familiar to children. The fourth design principle is that modularity, adjustability, and extendibility are inherent to task design. For example, an enumeration task may include placing eight identical bottle caps in a row. This task can be adjusted by using a different amount of caps, or by placing the caps in a different configuration (such as in a circle with no obvious beginning or end), by varying the color of the caps, etc.

The following excerpts are taken from sessions of our professional development program which focused on enumeration tasks that may be used in preschool. The participants were all practicing preschool teachers who were teaching 4–6 year old children in municipal preschools. The excerpts demonstrate how teachers became familiar with the process of task design and how they collaborated with the instructor in designing tasks to implement with young children.

I (Instructor): According to the curriculum guidelines, by the end of kindergarten a child should be able to count 30 objects ... we first ask him to count without giving him objects. Later on, when counting objects, we should think of how many objects to place before the child to count.

(Teachers offer different amounts.)

I: I probably shouldn't start with 30 because he may know how to count but the large amount can make it difficult. How about 10 items? Why isn't a good idea to start with 10?

T1: It's a large number.

T2: Because we have 10 fingers.

T3: Automatically, they say 10.

I: Right. How about 8? (The instructor places 8 identical bottle caps on the table.) Many times, a child expects there to be 10 so he won't necessarily take care to point to each one at a time. Instead, he might run his finger quickly over the items saying the numbers from 1 to 10. (The instructor demonstrates this.)

In the above segment, we see how the instructor first reminds the teachers of the curriculum requirement and then they discuss together the amount of items to be counted. It is also important to note that in designing the task, possible children's actions are taken into account. Knowing that ten is a benchmark number for children, and knowing that children may automatically count until ten regardless of the number of actual items to be counted, guides the instructor and teachers in choosing a different amount of items.

It is also important to consider the types of items to be counted. The instructor points out that if the items are of two colours, the child may count each colour group separately. She recommends starting by having the children count a set of homogenous objects such as bottle caps, easily accessible to the teachers and then afterwards checking what happens with heterogeneous items. In other words, the number and types of items to be counted are explicitly discussed features of this task.

Another task feature explicitly discussed with the teachers is the wording of the questions or the instructions they will give children.

I: What should we ask the child?

T6: (We should ask the child...) "How many are there?"

I: So, if the child points to each item and counts 1, 2, 3 till 8 then we know that he has the one-to-one correspondence principle. But do we know if he understands the principle of cardinality? So, after he counts, we will ask again, "How many items are there?"

T7: The first time we ask him how many there are, we are essentially requesting him to count the items. It's instead of saying, count the items. The second time we ask, "How many are there?" we are addressing the cardinality principle.

I: Correct. Some children will begin to count again from the beginning. And if we ask them again how many items there are, they will probably start counting again from the beginning. But, some children do not have the one-to-one correspondence principle; they say the counting sequence correctly while running their fingers over the items (the instructor demonstrates this action) but if they end at 7 or 9, when asked again how many there are, they will say whatever number they end up with. They understand the principle of cardinality.

In the above segment, we see how the different principles of enumeration are addressed by separate aspects of the task. First, the issue of reciting the number sequence without reference to items is mentioned. Then, the one-to-one principle and the cardinality principle are dealt with separately.

One of the ways we encourage teachers to think about task design is by having them design tasks with accompanying scripts which will focus on one specific enumeration principle. They then act out the proposed task, with one teacher playing the role of the preschool teacher (PT) and a second teacher playing the role of the child (PC).

PT places 8 different coloured disks on the table and arranges them in a circle.

PT: How many disks are there?

PC (counting around and around): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18.

PT: How many are there?

PC: 18.

PT: You counted the disks in the shape of a circle. Can you arrange them in a different shape?

PC places the disks in a row and proceeds to count correctly.

After the act, the instructor and teachers discuss what happened.

I: The circular arrangement is very special. The difficulty is that it promotes repetitions. Because there is no sign of who is first and who is last... The choice of which is first is arbitrary.

PC: If you notice, we purposely used disks of different colours. We could have used disks that were all the same colour and then...

PT: it would have been more difficult.

I: Correct. Having different colours may help the child remember where he started to count. Counting items in a circle requires knowledge of all the enumeration principles and then adds a degree of complexity... A circular arrangement allows us to see if the child feels the need to avoid repetition and if he feels this need does he have the skill to avoid it. This is a classical case where using identical items increases the difficulty of the task.

Finally, we point out that teachers chose several tasks to implement with individual children in their preschool classes, video-taped the task implementations, and discussed the experience during program sessions. The teachers agreed to implement the same tasks in the same way in order to be able to compare results of their efforts. These tasks are listed below:

1. Ask the child to count aloud till 10.
2. Arrange 8 identical items (bottle caps) in a row on a table and ask: (a) “How many (caps) are here?” (b) After the child finishes to count ask again, “how many are here?” (c) Point to the last cap counted by the child and say, “A different child said that he can start to count from here. Is he correct? Is this allowed?” If the child starts to count again say, “Just tell me if you think it’s allowed and why.”
3. Take the 8 items on the table and bunch them together. Ask, “How many are there?” If the child starts to count, stop her and say, “Can you tell me how many there are without counting?”
4. Arrange 7 identical bottle caps in a circle and ask, “How many are there?”
5. Place 30 items (bottle caps) on the table (in no particular order) and ask, “How many are there?”

The teachers’ final assignment for the course included implementing these tasks and then analyzing their students’ conceptions. In addition, teachers were asked to reflect on how they might change the task as a result of their experience. These reflections are the source of our data, and are discussed in the next section.

## 2 REFLECTIONS AFTER IMPLEMENTING ENUMERATING TASKS

This section begins by reporting on two teachers’ reflections, chosen because of their detailed reflections. We then present additional ideas for changing the task design raised by some of the other teachers.

### 2.1 FERN AND YOLANDA

Fern (all names are pseudo names) implemented the enumeration tasks with a  $4\frac{1}{2}$  year old girl, Gale. Before sitting with Gale she estimated that the child would be able to count aloud till 10 and enumerate up to 10 items. She was not sure if Gale would be able to enumerate items arranged in a circle or enumerate a large amount of items. After implementing the tasks, Fern reported that Gale skipped the number 6 when counting aloud. Gale correctly enumerated the 8 items, repeated that there were 8 items, agreed that you may count from either direction, and that even when bunched up, there remain 8 items. When shown 7 bottle caps arranged in a circle, Gale did not count but just looked at the caps and said that there were five. For the last in the series of tasks, Fern decided to place 20 caps on the table, instead of the agreed upon 30 “so as not to cause confusion and add stress.” Gale counted correctly up till 15 and then said 17, 18, 21, and 22. Fern then asked her, “so how many are there?” to which Gale answered, “22.” Fern thus concluded that Gale understands the cardinality principle.

When reflecting on the way she implemented the tasks, Fern considered the wording she used with Gale,

I felt that I was saying ‘enumerate’ so Gale stopped, that is, she is less familiar with this term than with counting so essentially it was my mistake because I usually use the term counting and not enumerating. I said to Gale ‘tell me how many bottle caps there are without enumerating’ and she was having difficulties with this, so I said, ‘without counting’ and she understood and answered.

Fern has several suggestions for changing the tasks. First, she would ask Gale to count aloud again in order to check her original assumption and make sure that Gale knows how to count. Second, she would encourage Gale to use her finger when counting items in a circle and if she still had difficulty with the task Fern suggests to make it easier by signalling a starting point with a different colour cap. Third, Fern states that “every task should be implemented twice in order to make sure what the child is able to do so that you can enhance the child’s confidence and self-efficacy”. Fern summarizes her experience, “the series of tasks allows me to know the child better, points of weakness and points of strength”.

Yolanda implemented the series of tasks with Tammy, a five-year old girl. According to her interpretation, Tammy knew how to count aloud till 10, enumerate 8 bottle caps placed in a row, and demonstrated the one-to-one correspondence principle and the cardinality principle. It was more challenging for Tammy to count the caps placed in a circle. Yolanda wrote that she implemented this task twice because,

... the first time, the girl answered correctly that there are 7 caps, but when enumerating the caps I saw that she touched two caps when saying the number 7 and so it ended up that she counted again the cap she started with. In other words, she didn’t remember where she started from... I felt that she simply was in a hurry and got mixed up and it wasn’t fair to conclude that she couldn’t complete the task. So, I gave her another opportunity and the second time she succeeded.

Tammy found it difficult to count 30 bottle caps. In consideration of this difficulty, Yolanda thought about how she might be able to help the girl and so she asked her, “Would you like to count the caps in a different way?” According to Yolanda, it seemed that this question was helpful. She wrote, “it was as if my question gave her permission to ask a question she didn’t have the nerve to ask beforehand: Can the caps be set up in another way?” Yolanda goes on to describe how Tammy arranged the caps in a circle and counted the caps demonstrating one-to-one correspondence up until 29 and then, instead of 30 she said 20 and 10. Yolanda concludes that she, as the teacher, needs to move Tammy forward from that point.

## 2.2 ADDITIONAL SUGGESTIONS BY OTHER TEACHERS

Goldie implemented the tasks with a 4-year old girl. The main challenge she encountered was counting 30 identical bottle caps. Goldie makes two suggestions for the future: she would either lessen the amount of caps to 20 or she would point out where to begin counting and arrange the caps in a more orderly fashion. She writes, “I understood that [the girl] at this age finds it much easier to count items when they are arranged in an orderly fashion and not spread out.” Dora also mentioned

a child who had difficulty enumerating 30 caps. She wrote, “The boy was able to count aloud in the proper order till 12. . . I would maybe have him try enumerating 12 items because that was the number he was able to correctly count to.” Similarly, Raley suggested first asking the child to count aloud till 30 before asking him to enumerate 30 items. Another teacher wrote that she would not change anything except that in the future she would use objects that the child likes to play with in kindergarten. Finally, Sabrina found that one child correctly counted the caps placed in a row, but did not understand what was meant by the question “can you count from this side.” She suggested asking instead, “Is there another way to count the caps?”

### 3 SUMMARY AND CONCLUSIONS

We begin by noting the details in teachers’ interpretation of children’s knowledge. They did not merely write that the child could or could not complete a certain task but attempted to analyze at what particular point the child showed strengths and weaknesses. For example, Fern noted that the little girl, Gale, enumerated correctly up till 15 but then mixed up the counting words. Fern differentiated between Gale not being able to say the counting words and knowing the cardinality principle. She also noted that Gale seemed to not understand the term enumerating and thus could not answer the question. Yolanda noticed that when Tammy counted the caps in a circle, she touched two together, but only did this once. This attention to detail echoes the design process experienced by teachers where emphasis was placed on knowing which part of the task was related to a particular knowledge construct or skill.

We see this same attention to details as teachers suggested ways of changing specific features of a task. Some of the teachers suggested they would change the number of items to be counted. Others said they would change the type of items or change the wording of a question asked. What is important to note is that not one of the teachers said that they would change more than one feature at a time. For example, Goldie claimed that she would lessen the amount of caps *or* point out where to begin. In a way, Goldie is varying the task in order to scaffold learning. The same may be said of Fern who suggested making a task easier by changing the color of one cap in order to signal the starting point. Likewise, Raley suggested that when counting 30 caps, it would help the child if the caps were not identical. These suggestions can be linked to the teachers’ participation in the design process.

What is also clear is that most of the teachers’ suggestions stemmed from what they learned about their students’ knowledge and difficulties. Dora made this connection explicit by writing that since her young student could count aloud till 12 she would have him enumerate 12 items instead of 30. The same may be said of the teachers who would implement a task twice because they could see that sometimes the child was just not focused the first time. A few teachers interpreted the situation as the task was ongoing and changed a feature while in the middle. For example, Fern and Yolanda changed the wording of a question on the spot when they realized that their young students did not understand the question being asked. But these changes did not change the essence of the tasks, which was to promote the use of one-to-one correspondence and the understanding of cardinality. Likewise, Fern decided to place 20 instead of 30 caps on the table so as not to cause the little girl stress. Considering that the little girl had another year of preschool ahead of



her, this change is both appropriate and thoughtful. These examples show how the teachers were keenly attuned to their students' engagement with the task, including affective aspects of engagement with the task, and made on the spot decisions about changing one particular feature that would help the student complete the task while preserving the essence of the task. Referring back to variation theory, we see that teachers related to some general variables of the task: phrasing (what exactly is said), representations (color and types of items to be counted), structure (what to do first, second, and repetition), and the child (age, knowledge, attention span). Changing one feature along one strand resulted in a new tailor-made task for a specific child for a specific purpose. All in all, we have demonstrated a chain relationship: knowing the design principles behind tasks and being aware of specific features of tasks impacted on what teachers learned from implementing those tasks which in turn impacted on how teachers changed specific features.

The question remains, would the teachers have noted the nuances of children's knowledge and would they have made and carried out deliberate and specific changes to task features if they had not participated in the design process? Or, expressed differently, was there something about the task design itself that might have supported these results without involving the teachers in the design process? Our tentative answer is a qualified yes. We feel that two of our design principles may be especially important if the design itself can lead to some of the results noted above. The first of these principles is that it should be clear which competency is being targeted at each point of the task and the second key principle is modularity. Regarding the first, clarity is in the eyes of the beholder. What may be clear to a designer may not be clear to a teacher. Thus, even if teachers are not involved in the design process, they should be encouraged to analyse existing tasks and attempt to connect task features to specific knowledge constructs and competencies they wish to promote. In fact, we have initial evidence of this possibility. Teachers who participated on our geometry program deliberately modified standard identification tasks which included only prototypical figures by adding non-prototypical figures in order to promote critical attribute reasoning. Regarding modularity, when a task is designed in such a way that it makes it simple for the teacher to vary one aspect while retaining others, the teacher can be flexible and think about what changes might be made as well as what changes should not be made in order to promote children's knowledge. We believe that a task designed with modularity can greatly influence teachers' adaptation of tasks and what they may learn about their students' knowledge. We qualify our yes by saying that even if a task is designed with both of the above principles, the teacher must be knowledgeable enough, both in mathematics and pedagogy, and in addition, have high self-efficacy, to make changes to a task that will promote children's knowledge. This is the on-going aim of professional development.

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