Peer-feedback as a part of collaborative problem-solving

Vrstevnická zpětná vazba jako součást spolupráce skupiny při řešení problému

Alena Hošpesová[1,*], Jarmila Novotná[2], Man Ching Esther Chan[3], David Clarke[3]

[1] University of South Bohemia, Faculty of Education, Jeronýmova 10, 371 15 České Budějovice, Czech Republic; hospes@pf.jcu.cz
[2] Charles University, Faculty of Education, Magdalény Rettigové 4, 116 39 Praha, Czech Republic
[3] Melbourne Graduate School of Education, Ground floor, 234 Queensberry Street, Melbourne, VIC 3010, Australia

The paper presents a study within an international project titled *The Social Unit of Learning*. This study examines the ways in which students interact with each other when solving an open mathematical problem in small groups of four. Attention is focused on peer feedback, especially on its form and how it affects the process of solving the problem. Our analysis suggests that feedback often appears in the group discussion and takes the form of both positive reaction (confirmation; question asking for clarification, request for clarification, eliciting) and negative reaction (rejection, explained or unexplained objection, doubt, question involving doubt). The analysis also identified a communication pattern that generalizes the course of the group discussion and factors influencing the quality of peer feedback related to the content structure of the problem (thematic, conceptual and competence layer).

Key words: collaborative problem-solving, peer feedback, mathematics education, 12 to 13-year-old students.

Received 12/2020
Revised 5/2021
Accepted 9/2021

Článek prezentuje výsledky výzkumné studie uskutečněné v rámci mezinárodního projektu nazvaného *The Social Unit of Learning*. Studie zkoumala způsoby interakce žáků při řešení otevřené matematické úlohy ve čtyřčlenných skupinách. Pozornost se zaměřila na identifikování forem vrstevnické zpětné vazby a jejího vlivu na proces řešení úlohy. Z analýzy vyplynulo, že se zpětná vazba objevuje v diskuzi skupiny často a má formu pozitivních (potvrzení; požádání o vysvětlení) i negativních reakcí (odmítnutí, vysvětlené nebo nevysvětlené námitky, pochybnosti). Analýza také identifikovala komunikační vzorce v diskuzi ve skupině a faktory ovlivňující kvalitu vrstevnické zpětné vazby vztahující se k obsahové struktuře problému (tematické, konceptuální a kompetenční vrstvě).

Klíčová slova: řešení úloh, vrstevnická zpětná vazba, výuka matematiky, základní škola.

Zasláno 12/2020
Revidováno 5/2021
Přijato 9/2021

1 Introduction, rationale

Learning is a social matter, which implies that knowledge and experience is gained through interaction, communication and sharing with others. Collaborative learning has been used as a pedagogical approach to enable students to solve challenging learning tasks (Damon & Phelps, 1989). It has been found to have strong effects on a range of dependent variables such as achievement, socialization, motivation, personal self-development (Gillies, 2016; Francisco, 2013; Isohätälä et al., 2018), and problem-solving skills (Lohman & Finkelstein, 2000). In the classroom, the promotion of collaborative learning facilitates students to take an active role in their learning through interaction with a group of classmates. The teacher gives instructions to the group of students in accordance with the aim of the lesson and further acts as a consultant and facilitator as the students process and complete the assigned tasks. In an ideal group situation, it is assumed that each student understands his/her role in the work and that the tasks assigned to the groups provide each student with opportunities to express themselves (Cohen, 1994; Cohen & Lotan, 2014).

The Social Unit of Learning Project investigated the social phenomena that characterise learning processes in a mathematics classroom (Chan, Clarke & Cao, 2018). The project used the *Science of Learning Research Classroom* in Australia and equivalent facilities in China to record audiovisually the interactions of whole classes of students and their teacher as the students engage in purposefully developed mathematical activities. In each filming session, multiple cameras and audio inputs were used to capture, in fine–grained detail, teacher–student and student–student interactions and speech throughout the session. A multi-theoretic research design (Chan & Clarke, 2019) adopted by the project involved the construction of a complex data set composed of video records and other supplementary data. The design allows an analytical team to examine different aspects of student collaborative problem-solving. An international multi-disciplinary research team (combining education, cognitive and emotive psychology, learning analytics, and neuroscience perspectives) was recruited to develop analytical frames for coding the data, including analysis of the negotiative foci of student exchange (Chan & Clarke, 2017); dialogic talk between students (Díez-Palomar et al., 2021); student agency (Nieminen et al., 2021); student motivating desires (Tuohilampi, 2018); student interactivity (Chan & Sfard, 2020); behavioural indicators of student engagement (Chan et al., 2020); student shared cognition (Clarke & Chan, 2020); and material resources (Moate et al., 2021). The researchers each constructed distinct data sets according to different theoretical perspectives applied to the same set of video records and other supplementary data. The multi-theoretic research design allowed the research team to juxtapose different interpretive accounts reflecting different theoretical positions in order to compare and contrast the capacity of different theories to characterise different aspects of the complex classroom setting; to examine their assumptions and implications, as well as their strengths and limitations.

When the authors of this paper started their cooperation, they decided to perform a didactic micro-analysis of communication within a group of students in the process of problem-solving. The research team originally thought that we would perform the analysis from the point of view of peer assessment. However, when working with the data, we found that the recorded reactions of students to suggestions of a classmate often do not have the nature of assessment. They are often a quick response to different types of messages. That's why we started using the term *feedback*, which is more concise. The results from the analysis are valuable for understanding the dynamics of working in groups and for gaining a better understanding of feedback mechanisms by peers when solving a problem. This can be of importance for teachers when preparing didactical situations based on group work to optimize social interactions and promote student learning.

2 Theoretical Background

2.1 Collaborative learning

Collaborative learning is an educational approach which aims to integrate academic and social learning experience in classroom activities (Gillies, 2016). Collaborative learning is not only arranging students into groups. Johnson and his colleagues (Johnson, Johnson & Holubec, 1994) stressed the social dimension in the successful incorporation of collaborative learning in the classroom through: positive interdependence, individual and group accountability, promotive face to face interaction, and the development of the students' interpersonal and small group skills, and group processing. Students must work in groups to complete tasks collectively toward academic goals. Unlike individual learning, which can be competitive in nature, students learning collaboratively can capitalize on one another's resources and skills (asking one another for information, evaluating one another's ideas, monitoring one another's work, and so on) (Chiu, 2008). Furthermore, the teacher's role changes from giving information to facilitating students' learning (Johnson & Johnson, 1984). Everyone succeeds when the group succeeds. Ross and Smyth (1995) described successful collaborative learning tasks as intellectually demanding, creative, open-ended, and involving higher order thinking. Collaborative learning has also been linked to increased levels of student satisfaction (Leikin & Zaslavsky, 1999, p. 244).

Hesse and colleagues defined collaboration "as the activity of working together towards a common goal" (Hesse et al., 2015, p. 33). They mentioned several elements included in this definition:

- communication (the exchange of ideas with the objective to optimise understanding by recipients accompanied by a division of labour);
- responsive contributions to planning and problem analysis;
- active and insightful participation.

Collaboration is dependent on factors such as a readiness to participate, mutual understanding, and the ability to manage interpersonal conflicts.

Collaborative learning as defined by Damon and Phelps (1989) can be seen as closely linked, if not synonymous with collaborative problem-solving. According to Hesse et al. (2015, p. 38), "collaborative problem-solving means approaching a problem responsively by working together and exchanging ideas." They emphasized its usefulness when dealing with complex problems, and a range of social and cognitive skills that students can acquire as part of the activity (e.g., perspective taking, social regulation, resource management, goal setting, among others). Research comparing collaborative learning with traditional instruction-based teaching showed that students who engaged in collaborative learning learned significantly more and enjoyed the classes more (Johnson, Johnson & Holubec, 1994). Leikin and Zaslavsky (1999, p. 245) examined the specifics of students' activities during successful collaborative learning in mathematics education. They indicated that the implementation of the exchange of knowledge promoted students' active explorations and lead naturally to an increase in students' mathematical communications. Students' attitudes towards such an exchange-of knowledge method were positive, and their achievements in the experimental method were at least as good as those of students learning in the conventional way. An investigation of the types of help those students received while learning showed that verbal explanation was the predominant type of help received by the students.

When analysing the communication of students participating in the *Social Unit of Learning Project*, we realised that one noteworthy part of the collaborative problem-solving activity is that the students provided their schoolmates with "feedback" during the activity, which sometimes was and sometimes was not evaluative in nature. This stimulated our curiosity regarding the ways in which students' collaborative learning activities constituted useful feedback for each other's mathematical learning. Our objective is to examine how the discussion in the group supports understanding and shifts or progresses the problem-solving procedure.

To further explain, feedback can be considered as a process of collecting information about people's reactions to a certain message (Žantovská, 2015). In this sense, feedback is used in many areas of human life, and it is not a concept specific to the field of education. In education, feedback is defined as "information provided by an agent (e.g., teacher, peer, book, parent, self, experience) regarding aspects of one's performance or understanding" (Hattie & Timperley, 2007, p. 81). Feedback is seen as integral to assessment that supports learning, that is, formative assessment (Black & Wiliam, 2009). In the following section, therefore, we start from the explanation of what we understand about formative assessment, and discuss the differences in assessment carried out by teachers and by classmates.

2.2 Formative assessment, self-assessment, and peer feedback

Assessment can be defined as the "process of reasoning from evidence" (Pellegrino, Chudowsky & Glaser, 2001, p. 43), and can be seen as a natural part of human activities. In educational settings, assessment can be distinguished in terms of the purpose for which the assessment is directed. *Summative assessment* "[describes] learning achieved at a certain time for the purposes of reporting to parents, other teachers, the students themselves and, in summary form, to other interested parties such as school governors or school boards" (Harlen & James, 1997, p. 370). *Formative assessment* informs teachers about "students' existing ideas and skills, and... the point reached in development and the necessary next steps to take" (Harlen & James, 1997, p. 369). It involves the processes of data collection and interpretation that teachers use to make decision about the following: What have the students learned? What is student's learning objective? How can students be supported and assisted during their learning?

Formative assessment has its basics in educational evaluation practices. Researchers in the United States, the United Kingdom, Australia, and New Zealand have contributed to the shift from a focus on formative evaluation to formative assessment or assessment for learning and brought this issue into the spotlight for teacher education programs, teacher professional development, and educational research. Formative assessment has been slowly introduced into the school practice in different parts of the world (OECD, 2005), including the Czech Republic (Žlábková & Rokos, 2013). In upper secondary education practice, different methods of formative assessment are used, for example: marking-grading, structured classroom dialogue, immediate teacher feedback as a response to a student's problem-solving (on-the-fly assessment) or less frequently written teachers' comments or formative peer assessment.

From the perspective of this research, the notion of formative peer-assessment is particularly relevant, because of the emphasis on peer interaction in the course of collaborative problem-solving. Peer-assessment is an educational approach for classmates to judge the level, value, or worth of the products or learning outcomes of their equal-status peers. Boud and Falchikov stated that (2007, p. 132):

> Peer assessment requires students to provide either feedback or grades (or both) to their peers on a product or a performance, based on the criteria of excellence for that product or event which students may have been involved in determining.

Peer assessment and the assessment made by the teacher, differ considerably in the purpose for which the data are collected and interpreted. In the process of assessment, the teacher has to make a number of decisions about: (a) the data to be collected (e.g., whether systematically or ad hoc), (b) interpretation of the assessment results, (c) communication about the results, and (d) building further decisions based on the results (Black et al., 2004). Peer assessment does not cover the full range of these intentions. The student reflects on the pieces of information obtained from the classmate(s), interprets them, and communicates about them with the classmate(s). The student may compare the classmate's solution with their own solution as a means of providing feedback. They interpret their classmate's solution intuitively, mostly without the intention of supporting someone else's long-term learning.

Despite differences between assessment by teachers or by peers, Slavík (2003) emphasized the importance of peer assessment which the students themselves use, manage, which they understand to the necessary extent and which they can explain or possibly defend. Slavík (ibid) theorised that autonomous assessment, which is in fact the aim of school education, can develop and deepen through self-assessment and through assessment of others' performance (most likely of classmates, i.e., peer assessment) where students learn to reflect on their work. Boud et al. (2010) stressed that students and teachers in higher education become responsible partners in learning and assessment. Students should progressively take responsibility for assessment and feedback processes and for that reason they should develop and demonstrate the ability to judge the quality of their own work and the work of others.

Peer-feedback can be effective in supporting student learning for several reasons:

– the mutual exchange of ideas and advice is realized in students' language, where the exchange is formulated in verbal phrases that students commonly use (Black et al., 2004, p. 14);

– if the students do not understand the explanation, it is easier for them to ask their classmate rather than address the teacher in such a situation (students may find the teacher's feedback too complicated, so they prefer peer language) (ibid);

– research and experience show that students can learn from each other even though their intellectual level is different; they share their assessment with each other and can thus better master the subject matter themselves (Sluijsmans, 2002).

These three reasons served as the basis for formulating the research questions.

2.3 Research questions

The general objective of our research was to verify the contribution of providing and receiving feedback from classmates in the process of collaborative problem-solving. We elaborate this objective in terms of two research questions:

What forms did peer feedback take during the communication between a group of students when solving a problem together?

In what way did the students' peer feedback influence the problem-solving process?

3 Data and methodology

3.1 Data collection

The data used for the analyses presented in this paper were from the *Social Unit of Learning Project* and were collected in the research classroom facility, the *Science of Learning Research Classroom*, at the University of Melbourne, Australia. The research classroom is a university teaching space with built-in video cameras and audio recording equipment for capturing the activities of teachers and students in the room. In the *Social Unit of Learning Project*, Year 7 students (12 to 13 years old) with their usual teacher were brought into the research classroom to participate in purposefully designed activities in mathematics. The students undertook a sequence of tasks individually, in pairs, and in small groups (typically four students). The conditions for student collaborative problem-solving and learning were manipulated so that student and teacher contributions to the learning process can be distinguished. For more details, see (Chan, Clarke & Cao, 2018).

All students' (and the teacher's) classroom speech were video- and audio-recorded and all students' written work was copied and scanned. The organization of the research classroom enabled the recording of the interactions of each student group. The video recordings of the group discussion were transcribed.

The students completed a mathematics problem as part of a larger project that investigated student collaborative problem-solving (for more details, see Chan, Clarke & Cao, 2018). Since the focus of the project was on student-student interaction, the teacher was given explicit instructions to minimise intervention in the students' problem-solving activity. In this paper, video recordings of the collaborative problem-solving activities of two groups of four students were analysed (Group Number 3 and 5). The groups were chosen on the basis of their written solution which appeared to be distinctly different compared to the other groups (see Chan & Clarke, 2017). These two groups then became the focal groups for the application of different analytical perspectives (see Section 1).

3.2 Solved problem and its analysis

The following open-ended problem (Fred's Apartment) was given to students verbally by the teacher, visually on the student worksheet, and was projected on the screen in front of the classroom:

Fred's apartment has five rooms. The total area is 60 square metres.

1. *Draw a plan of Fred's apartment.*
2. *Label each room, and show the dimensions (length and width) of all rooms.*

The students were given 20 minutes to complete the open problem. Silver (1995) described several types of open problems and explained that they are "susceptible to different interpretations or to different acceptable answers" (p. 68). This definition fits the Fred's Apartment problem. Although the educational goal of the activity was not explicitly stated, in the recorded lesson, we can deduce that the students should deepen their problem-solving competencies. The problem was purposefully designed so that students need to draw from knowledge outside of the classroom and determine the task constraints themselves in order to complete the task.

In analysing the structure of educational content, Slavík et al. (2017, p. 242) distinguished several layers of educational content of a task – see the diagram in Fig. 1.

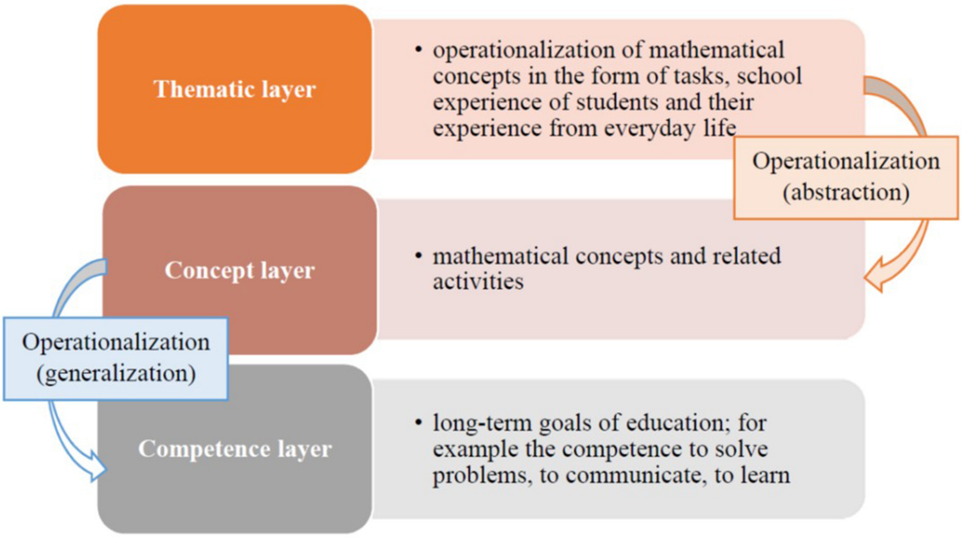

Fig. 1: Diagram of the analysis of the structure of educational content (adapted according Slavík et al., 2017, p. 342)

Analysing the problem of Fred's Apartment, we start from the conceptual layer of the problem. We identify which key concepts students have to master in order to be able to solve it. The central concept in the task is the concept of area (how it is measured, calculated and what it means). Related to this is the shape and dimensions of the shape in the plane, which we know has the area of 60 m^2. In addition, this shape should be divided into five sections, for which students choose the shape and dimensions.

When designing the shape of the apartment, its dimensions and division into parts, students rely on experiences from everyday life in terms of what they know about living spaces – they move to the thematic layer of the problem.

To understand the following analyses, we include a brief summary of how the groups proceeded to solve the task.

Group 3 (two girls: Audrey and Katie; two boys: Pedram and Poya) started solving the problem by considering the size of the rooms. The students first solved this problem arithmetically – how 60 square metres can be divided into 5 parts. The discussion was confused by the fact that some students were talking about the area of the rooms, others about their dimensions. From the beginning, this group tried to decide whether the corridor should be part of the apartment. They repeatedly returned to the issue and referred to various practical experiences (for example the size of the room, the accessibility of the flat from outside). The group also addressed the issue of door placement and movement around the apartment. Finally, in the last two minutes, a student (Katie) drew a plan (Fig. 2) of the apartment on her own. The others gave her no feedback.

Group 5 (two girls: Anna and Pandit; two boys: John and Arman) started solving the problem by discussing the scale of their drawing. The discussion was rather lively but it focused on the form of the flat/rooms, their sizes, number of rooms etc. Group 5 was reminded by the teacher that their task is to find the best solution, not the only existing solution. The discussion in Group 5 covered the size of rooms if rooms on the plan were proposed, the number of rooms and their types (kitchen, toilette, living room, lounge room, missing bedroom, workplace, . . .). All the time, the discussion focused more on practical aspects than a mathematical solution. When Group 5 started to construct a plan, several advantages and disadvantages of different proposals were raised. Social relationships in the group penetrated through

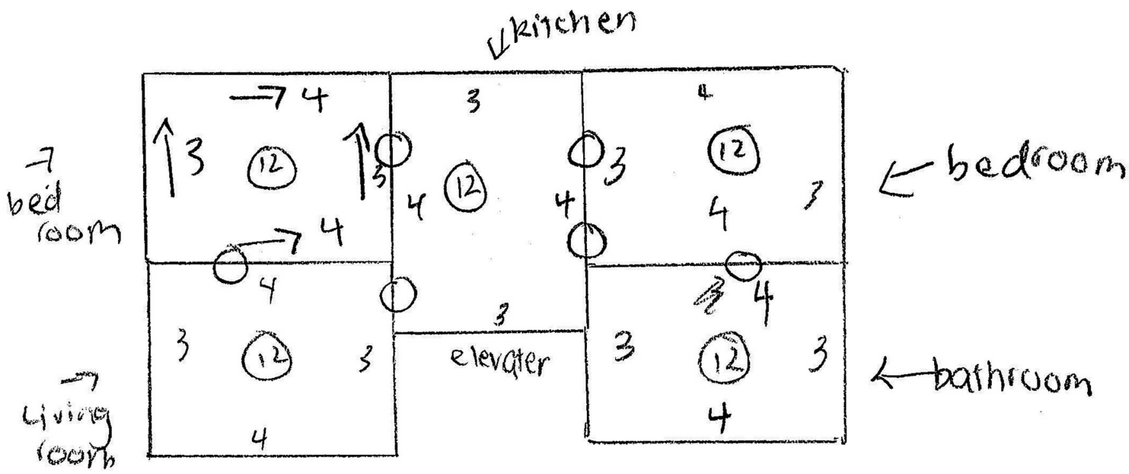

Fig. 2: The solution to the problem drawn by Group 3 (Reprinted from Chan & Clarke, 2017, p. 957, with permission)

the whole activity. There were different attitudes towards mathematics in Group 5 (Tuohilampi, 2018). Finally, the plan in Figure 3 was drawn but even at the end, not all group members were fully happy with the plan. The group did not mention a corridor, but it is included in the plan. The area of 60 m^2 covers the whole flat including the corridor and sanitary facilities.

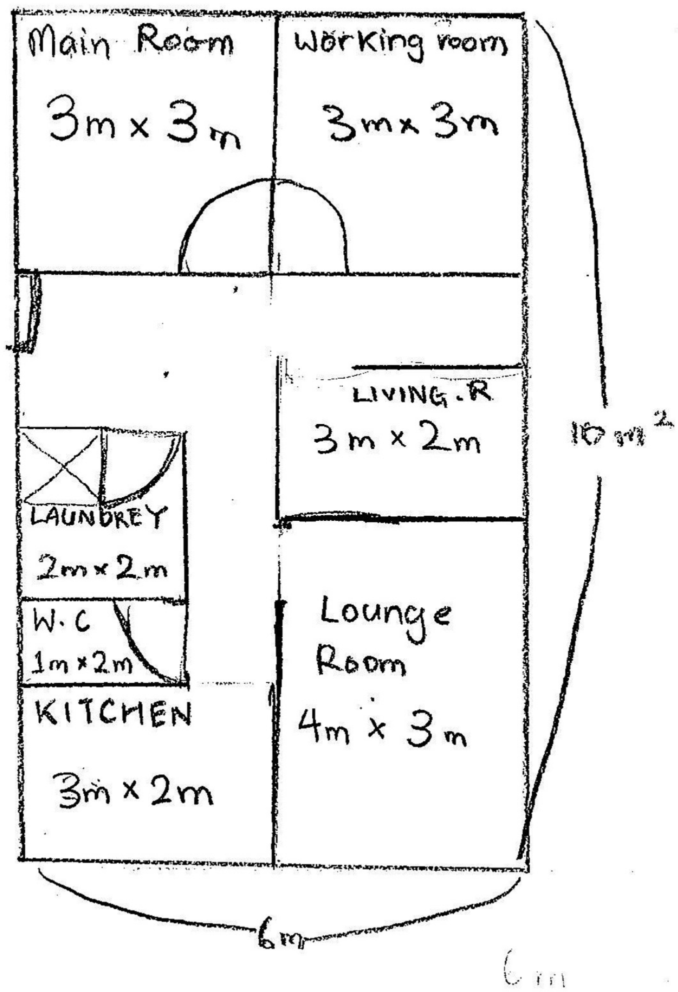

Fig. 3: The solution to the problem drawn by Group 5 (Reprinted from Chan & Clarke, 2017, p. 958, with permission)

3.3 Search for the aim of the research

Hošpesová and Novotná were offered the opportunity to analyze data obtained by Chan and Clark (members of the project team of *Social Unit of Learning Project*) from their research perspective in 2018. At that time they were focusing on formative assessment, namely peer assessment in mathematics. They realized that certain parts of communication of pupils in a group can be regarded as formative peer feedback. The team of authors of this paper decided to conduct a descriptive case study (Hendl, 2005) and formulated the first research question: What forms did peer feedback take during the communication between a group of students when solving a problem together?

In searching for an answer to this question, the team increasingly found that moving towards solving a problem was complicated for the students in some cases. The second research question was formulated: In what ways did the students' peer feedback influence the problem-solving process?

3.4 Determining how to analyse the data for peer feedback

The research design of the current study was qualitative. This was due to the nature of the research questions and the data we analysed. The research had the character of a case study (Hendl, 2005) examining the communication of members of a group of students aimed at solving a mathematical problem. We were inspired by grounded theory research design (Strauss & Corbin, 1990) in our analytical process. Through coding and analytical processes, we identified peer feedback as the main phenomenon for supporting the solution process.

Data analysis was conducted in two delimited stages. The search for the answer to the first research question started by open coding. At first, we worked independently. The research team members processed the transcripts of students' communication in both groups using open coding. The research team exchanged conclusions from analysis during the personal meetings and via Skype communication and looked for consensus in interpretations.

In the next phase, the research team members analysed the transcripts of both groups of students again independently using the list of codes that were agreed upon.

At the joint meeting, we sought re-occurring codes to search for communication patterns in the groups' discussion.

During the analytical procedure we examined segments of communication. Each segment contains a sequence of group members' statements discussing an issue initiated by a group member. This is consistent with Clarke's (2001) definition of a *negotiative event* as "an utterance sequence constituting a social interaction with a single identifiable purpose" (Chan & Clarke, 2017, p. 959). We concentrated on those negotiative events in communication related to problem-solving. We omitted the events which had the nature of a "social" conversation that was not task-focused. The following steps were then followed:

- Selection of negotiative events to be analysed.
- Analysis of students' interaction.
- Comparison of analyses of negotiative events and looking for more general phenomena characteristic for collaborative learning when solving this problem.

In the discussion of Group 3 we distinguished 18 events, and in Group 5 there were16 events.

4 Results

4.1 What forms did peer feedback take during the communication between a group of students when solving a problem together?

In the list of codes created during open coding, we identified various topics, such as: who carried out the peer feedback, what is the form of peer feedback, how does the assessed person react, and what is assessed. In the following discussion, we concluded that the feedback provided within the student group appeared to be a key factor in influencing learning. Based on the team's experiences, we constructed the initial version of codes labelling different forms of peer feedback and the students' reactions to the feedback.

Forms of peer feedback:

- Agreement (with or without any explanation)
- Objection (without explanation, explained)
- Question
- New proposal

- Praise (of the assessed person, of the assessed solution)
- Doubt (statement, question addressed to schoolmates)
- Change in the solving process
- Self-critique (of mathematical procedure, of the own consideration)

Reactions of the assessed person:

- Changes the result
- Clarifies or refines the original idea
- Restarts the solving process from the beginning (the same or a different one)
- Returns to the previous step
- Stops working/communicating

These codes allowed us to describe the on-going communication in the group. However, they did not indicate much about the individual students' contribution to the solution of the problem.

4.2 "Messages" in peer-feedback

As we went through the transcripts repeatedly, we realized that in the communication of a group of students' utterances were of a similar nature. During ongoing comparison of events and a search for a pattern, it was revealed that there was a similarity with the ESRU model. The ESRU model was discovered by Ruiz-Primo and Furtak (2006) during the analysis of a teacher's feedback on a student's idea that was unexpected. The acronym for this model is derived from the initials of the names of four consecutive steps in providing immediate feedback:

1. The first step is to invoke an interaction (called "E" from *eliciting*).
2. The second step is the *student's response* (called "S"). Here the student responds to the teacher's question, opinion, or objection from the previous step.
3. The third step is the *recognition* (marking "R") of the student's reaction or the teacher's response to the student's reaction.
4. The fourth and probably the most difficult step for teachers is the *use* of the information obtained (marking "U").

We believed that coding, which followed the steps of this model, would help us find the answer to the second research question. We entered the second stage of the research. We used only four codes from this model in the coding process: eliciting, response, recognition, use. Again, we worked independently at first and compared our conclusions in our joint "sessions". In addition to searching for a pattern, we also paid attention to the content of students' communication, to be able to answer the second research question.

The communication, however, was naturally not as "straightforward" as in the communication controlled by the teacher.

1. In a group of students, the discussion of an idea (negotiative event) started with *eliciting* provided by one of the students. Eliciting took various forms, for example the student turned the attention of classmates to some information in the assignment, proposed an idea (next step(s) of the solution), or simply the student asked a question.
2. One (or more) student(s) gave *feedback* to the proposal. As feedback, we will interpret all the reactions of classmates to the proposal. This reaction took the form of an admission, a question, a request for clarification. It could also be a rejection.
3. The one who elicited the discussion *responded* to the feedback from his/her classmates by trying to modify or refine his/her idea as required. Sometimes he/she gave up and suggested a new idea. Sometimes he/she just repeated the original idea. We did not label this reaction as recognizing, because it has a different nature than the recognition given by the teacher. We label this step as *reaction*.
4. If the one who elicited the discussion pushed through the idea, it is used in the next discussion, or in solving the problem or task. Here it seems apt to stay with the original designation: *use*.

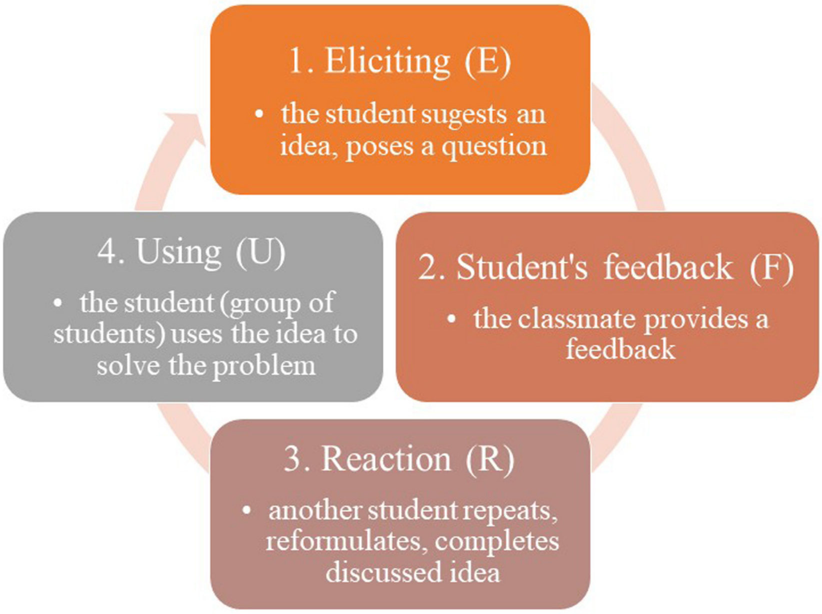

Fig. 4: Model EFRU and its components (schema adapted for this study)

The structure of communication can be described as an EFRU model and it is schematically illustrated in Fig. 4.

In practice, of course, some steps were found to be repeated. Also, in our data, there were several times where the student discussion did not show a clear intention (is the idea applicable to solving the problem or not) and the group reacted to the eliciting response of another student in the group. The group also repeatedly returned to some ideas (e.g., whether a corridor can be considered as a room or not – see below) (schematically elaborated in Fig. 5).

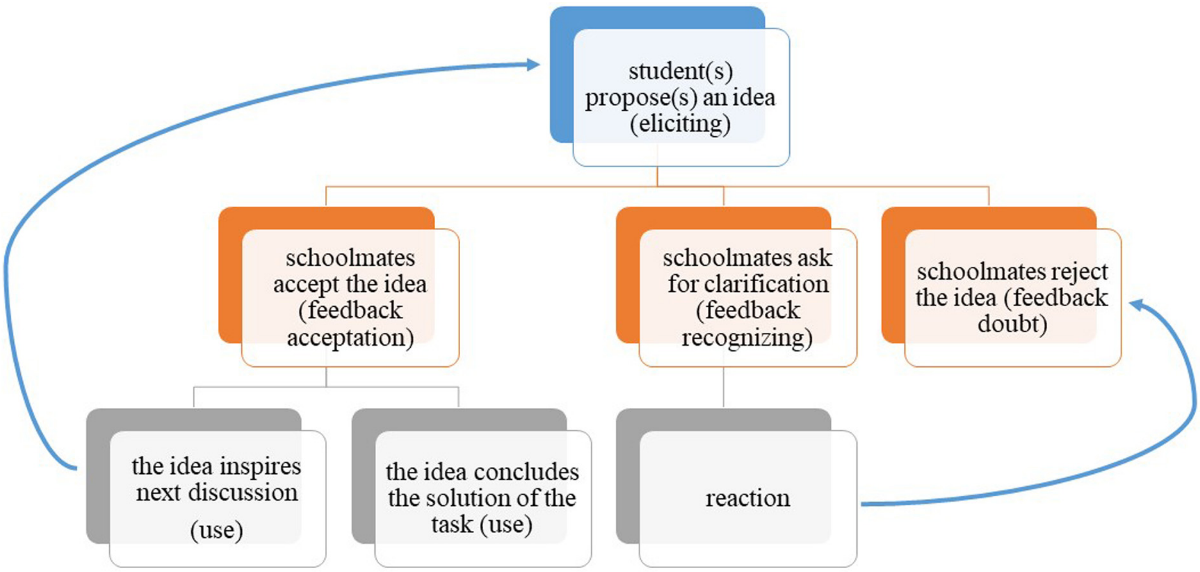

Fig. 5: Application of the EFRU model in peer feedback during group discussion (our own scheme)

The above model is illustrated in the example communication of Group 3 (Transcript 1). Here you can see the cyclic character of the communication (how some of the steps were repeated).

In the performed analyses of students' discussions, the EFRU model appeared to be a suitable tool to deal with the complexity of the student discussion enabling us to take into account the significance of individual utterances for the solution of the problem.

Tab. 1: Transcript Excerpt 1: The communication when clarifying the problem "corridor" (group 3)

Time	Dialogue	Code
101 [36:14.11]	Pedram: But how large will the rooms be?	Eliciting (question)
102 [36:16.24]	Poya: Twelve metres squared.	Eliciting (answer)
103 [36:18.08]	Pedram: Twelve metres squared so there's no corridor. (Over speaking)	**Feedback (objection)**
104 [36:22.00]	Poya: Well make them eleven metres squared.	Reaction (changes original idea)
105 [36:23.10]	Pedram: If there are 12 metres square and there are five rooms... Dude listen, if there are 12 metres square rooms and there are five rooms, that's the whole 60 metres	**Feedback (explanation of the objection)**
106 [36:33.06]	Poya: Okay. Maybe 11 metres squared.	Reaction (changes original idea)
107	Pedram: So there is only five metres.	**Feedback (objection)**
108 [36:40.00]	Poya: Six, really, six metres squared corridor.	Reaction (clarification)
109 [36:40.12]	Katie: There's only five rooms.	**Feedback (doubt)**
	...	
135 [38:09.11]	Pedram: Sixty metres, make each room five metres. I mean five – each room five metres?	Eliciting
136 [38:21.00]	Poya: Ten metres.	**Feedback (new proposal)**
137 [38:23.02]	Pedram: Five square metres. Are you smart?	Reaction (explanation)
138 [38:26.21]	Poya: Maybe.	**Feedback (doubt)**
139 [38:27.11]	Pedram: Five metres, you can't really step in the room because it's five metres with the walls. Not the floor, you idiot.	Reaction (recognizes of own mistake)

4.3 In what way did the students' peer feedback influence the problem-solving process?

The feedback that the students provided to each other is very frequent (see example in Transcript 1), but brief, and sometimes formulated imprecisely. However, the students seem to understand the meaning of each other. For example: at the beginning of their exchange in Transcript 1 (turn 105) Pedram points out that if the five rooms are each 12 metres squared, that they will take up the total area of the apartment (60 square metres) with no space for any corridors. Poya suggests an adjustment by saying: "Okay. Maybe 11 metres squared." Despite the brief statement, Pedram seems to understand what Poya means. Without explanation, he says the size of the corridor will be five (square) metres (turn 107). However, the correct unit is not used. Poya's reaction ("Six, really, six metres squared corridor" turn 108) is confusing and Katie's feedback ("There's only five rooms.") recalls the task instructions and, in our interpretation, shows that Katie probably doesn't understand her classmates. The group returns to the idea of dividing the area of an apartment into five rooms of the same size a little later by Pedram's proposal (turn 135), which is, however, incomprehensible. He seems to have confused the number of rooms (five) with the area of the room. Poya continues with his original idea, proposing to reduce the size of the rooms to 10 metres (squared) (turn 136). Pedram repeats his original idea (turn 137). Poya does not insist on his idea, and expresses doubt about himself being smart (turn 138). Pedram realizes that their solution is not realistic. The negotiative event can be summarized as: the boys clarified their thoughts, they actively participated in solving the problem. Poya's idea might have led the group to solve the task, but in the end, he was uncertain about his classmates' reactions and abandoned the idea.

In the second stage of the research we tried to reconstruct the reasoning of individual students on the basis of what they had said in the discussions. Although we are aware that we may have interpreted the students' intentions incorrectly, we believe we have managed to identify the moments when the group discussion was moving an appropriate solution forward as well as moments that were clouding the solution.

Peer feedback shifting the problem-solving ahead

As previously outlined, peer feedback may be beneficial for helping students in the process of solving the problem as compared with students who were solving the problem on their own. An advantage of peer feedback over teacher feedback is that students have a better understanding of each other. As students use "similar language", they are able to mutually clarify the meanings (Kaur et al., 2013). For example, communication transcribed in Excerpt 2 leads to a learning moment when Pedram realised that square metres do not mean that the rooms in the apartment need to be squares (turn 208). Although the

Tab. 2: Transcript Excerpt 2: Example of a dialogue when the peer feedback contributed positively to the problem-solving process (Group 3)

Time	Dialogue	Code
201 [41:08.10]	Pedram: Are we going to have square rooms though? Oh yeah, it's square rooms.	Idea (eliciting)
202 [41:11.02]	Katie: Do you say one room is 12 metres, one off side of the wall would have to be three, the other would have to be three and four and four. He said square metres.	**Feedback (ask for clarification)**
203 [41:19.00]	Pedram: No, they all have to be three. You can't get 12. That'll be a rectangle. Like you can't have one side of the length and ones on that side of the width. They've...	**Feedback (clarification)**
204 [41:33.16]	Pedram: If you're having a square room all of the sides are going to be the same.	Eliciting – idea
205 [41:37.13]	Katie: Does it have to be a square?	**Feedback (doubt)**
206 [41:39.11]	Pedram: Yeah. Square metres.	Eliciting – idea
207 [41:42.17]	Katie: Square metres mean like a metre inside the square. It doesn't mean that the room has to be a square.	**Feedback (explanation)**
208 [41:47.08]	Pedram: Yeah. Can be a rectangle.	Reaction
209 [41:48.16]	Katie: Yeah.	Reaction (doubt)
210 [41:49.05]	Pedram: But should it – well isn't it easier if it's a square?	**Feedback (doubt – question addressed to schoolmates)**

feedback that Katie gradually provides (turns 202, 205, 207) was not easy to understand, it eventually makes Pedram aware of an important idea that will remove the obstacle that complicated his process of problem-solving.

In our data we observed that it was difficult for the students to follow the problem-solving process, while assessing the contribution of peers. In the example of communication (Transcript Excerpt 3), two subgroups were created. The girls (Pandit and Anna) tried to solve the dimensions of the whole apartment (turns 301–307 and 310, 311, 316–318). Turn 318 was clearly inspired by a classmate's suggestion. The idea of the girls was then used by the whole group. Meanwhile, the boys (John and Arman) did not follow the girls' exchange and discussion of the area of each room.

Tab. 3: Transcript Excerpt 3: Peer feedback (Group 5)

Time	Dialogue	Code
301 [40:36.19]	Pandit: No. Wait, isn't that – have to times?	Eliciting – question
302 [40:38.18]	Anna: Yeah.	
303 [40:39.16]	Pandit: Twenty times thirty is like 600.	Eliciting (development of the previous idea)
304 [40:41.14]	Anna: Six hundred.	**Feedback (confirmation)**
305 [40:42.17]	Pandit: It has to be 60.	Reaction (confirmation that the solution is wrong)
306 [40:43.11]	Anna: Yeah. (Sigh)	**Feedback (confirmation)**
307 [40:45.12]	Pandit: You did it wrongly, that's why.	Reaction.
308 [40:45.12]	John: The area of the ... of each room is the same or not same?	Eliciting (question)
309 [40:46.20]	Arman: John. Fifteen times four is 60 and 15 times five is 75.	Eliciting (idea)
310 [40:51.09]	Anna: I don't know.	**Feedback (doubt)**
311 [40:51.18]	Pandit: I don't care about that.... No, wait. Wait.	Reaction
312 [40:53.17]	John: We need care about that – because maybe some rooms are bigger and some rooms are smaller.	**Feedback (confirmation)**
313 [40:56.20]	Arman: Fifteen times five is 75 and 15 times four is 60.	Reaction
314 [41:00.08]	John: Huh? What?	**Feedback (doubt)**
315 [41:01.20]	Arman: Fifteen times four is 60, but we need five rooms. Five, five.	Reaction
316 [40:57.19]	Anna: Twenty, three. Twenty- no, no, no.	Eliciting (idea)
317 [40:58.24]	Pandit: My god, you do like 10 times six or something like that.	Eliciting (idea)
318 [41:01.18]	Anna: Okay. Let's just do 10 times six then.	**Use**

Sometimes although the peer's eliciting was relevant for the solution and the schoolmates reacted to it for a certain period, they did not finish the discussion, or make use of the idea, but shifted to dealing with something else. For example, during the discussion in Transcript Excerpt 4 for Group 3, several suggestions were made about the corridor in the apartment. In the end, the students concluded, that there would be no corridor in the apartment. Pedram was not satisfied with the result and the students' discussion was interrupted.

Tab. 4: Transcript Excerpt 4: Peer feedback (Group 3)

Time	Dialogue	Code
401 [50:39.23]	Pedram: . . . Like how do you get out of the house?	Eliciting (question)
402 [50:42.05]	Poya: You need a back door.	Eliciting (idea)
403 [50:43.11]	Katie: There's doors everywhere.	Reaction
404 [50:43.23]	Audrey: Elevator.	Eliciting (idea)
405 [50:44.24]	Katie: The circles are the doors. Like you could label more doors if you want. (Laughter)	Reaction
406 [50:50.19]	Pedram: Okay	**Feedback (agreement)**
407 [50:51.13]	Katie: Just make sure there's doors –	Eliciting (idea)
408 [50:53.08]	Pedram: But there's no corridor in the house.	**Feedback (objection)**
409 [50:55.15]	Poya: No. (Over speaking)	Reaction
410 [50:56.04]	Audrey: There doesn't need to be a corridor.	Reaction (use)
411 [50:57.08]	Pedram: How to get out? Why don't you get lost?	**Feedback (doubt)**

4.4 Solved problem and educational objective of the activity

As already mentioned, the task did not have a specifically stated mathematical goal. The teaching situation was directed to the competence layer, where it supported the instrumentalization of the problem solver's experience (or mathematization) and the competence to solve problems.

In our view, uncertainties in the thematic layer of the problem caused difficulties for Group 3 in the solution process. The communication of the students repeatedly returned to the problem of whether there should be a corridor in the apartment or not. Even in real life, the question of whether the area of the corridor is a part of the whole apartment area differs according to the purpose of the calculation. The group did not manage to solve the question of the corridor in a satisfactory way. An objection to count the corridor as a room was raised multiple times during the student discussion. Nevertheless, they did not reach an agreement. The communication stagnated and did not progress the problem's solution. Transcript Excerpt 5 shows the student's helplessness with the solution of a non-mathematical content

Tab. 5: Transcript Excerpt 5: Non-mathematical problems in communication (Group 3)

Time	Dialogue	Code
501 [46:36.22]	Katie: Okay, fine. If you – if you do this, you can have a back garden. You can just walk out of your back garden to the front. (Laughter)	Eliciting (idea)
502 [46:43.12]	Pedram: Why do we even need a garden?	**Feedback (request for clarification)**
503 [46:46.13]	Audrey: It's an apartment though. That's okay.	**Feedback (objection)**
504 [46:48.05]	Pedram: That can – that can even be. . .	
505 [46:49.17]	Katie: Oh wait, it's an apartment.	**Feedback (objection)**
506 [46:50.17]	Pedram: Right.	Reaction (positive)
507 [46:51.17]	Katie: Oh my God, I forgot it was an apartment.	Reaction (awareness)
508 [46:54.13]	Pedram: Wait, you know what we can have? We can – we can bike it downstairs.	Eliciting (idea)
509 [46:59.15]	Katie: Mmm hmm.	**Feedback (positive)**
510 [46:59.18]	Pedram: Oh it's an apartment.	**Feedback (confirmation)**
511 [47:01.03]	Katie: Yeah. So you don't – you can't get out of the house. Technically, you have to take an elevator down.	Idea (eliciting)
512 [47:04.24]	Pedram: You – you do have to get out of the house.	**Feedback (objection)**
515 [47:10.18]	Pedram: . . . if you can't get out of the house. Just go to prison. They give you free food and free beds.	Idea (eliciting)
516 [47:18.02]	Poya: You go – you go – you – you think you go to jail if you don't get out of the house.	**Feedback (confirmation)**

of the task. The ideas formulated in turns 501, 508, 511, 515 are unrealistic. The students formulated the feedback in the form of objections. Ambiguities in the thematic layer of the problem seemed to have caused some negotiative events to become a barrier to progressing the solving process.

Nevertheless, the group has produced a sketch in which there was no corridor (Fig. 2).

5 Conclusions and discussion

This section outlines the contributions and limitations of this study to research into student's peer feedback as formative assessment and for problem solving. We first discuss the appropriateness of the EFRU model for describing patterns in students' feedback during problem solving, and evaluate its strengths and weaknesses. We also discuss the benefits of group cooperation in solving problems.

5.1 Strengths and weaknesses of the EFRU model

We propose an EFRU model (eliciting, feedback, reaction, use) as a general pattern of students' communication in a group and the feedback they provide to each other when solving a problem. This model allowed us to describe the course of communication in a group of students, but also to get an idea of how the feedback from classmates helps students to think during the process of solving the task. This is because the model itself also requires determining the intention of the speaker. This means not only describing what the students are saying, but also determining how they are thinking. When coding communication transcripts, we followed how individual students contributed to the communication, and we watched the thinking manifested in their comments. We are aware of at least two weaknesses in this approach. First, the students certainly did not communicate all of their thoughts to their classmates. But we can assume that they communicated what they considered as important. Second, our interpretation of what the student thinks is a postulation that we made when re-reading the communication, viewing the video, and based on the student's written products. It is a postulation that was supported by various types of data. We believe that in future research, these weaknesses could be addressed through post-lesson interviews with students. This approach has been used in the Learner's Perspective Study methodology (in more details in Kaur et al., 2013). But even in that project the success of this approach has not been unequivocally confirmed, because in some cases it was not possible to establish good contact with students.

5.2 Forms of feedback and its influence on the problem-solving process

Feedback appeared very often in the discussion of a group of students and takes the form of positive and negative reactions. In our analyses, we found the following forms of peer feedback:

- negative reaction: rejection, explained or unexplained objection, doubt, question involving doubt;
- positive reaction: confirmation; question asking for clarification, request for clarification, eliciting.

It is not possible to conclude unambiguously from our data that peer feedback helps or does not help to progress the solving process in a productive way. As it was shown in part 4 of this paper, the approach to peer feedback is not consistent across groups or within groups. In some cases, peer feedback resulted in moving the solving process ahead, in other cases this effect did not occur. In our data, the latter was mostly caused by constrained interpersonal relationships among group members. For example, by attempting to work individually without listening to the ideas of others. Willingness and ability to cooperate is not a characteristic of everybody; in a case that the willingness to listen to others' ideas is missing, the peer feedback is unable to alter the student's thinking or actions.

Using a similar language is often considered a great advantage of collaborative work in the groups but there could be inaccuracies in the way in which ideas are expressed. In Excerpt 1 and our comments to it, we have already shown that despite these ambiguities, students seem to be able to understand each other. However, sometimes inaccuracies in language influence the solution process negatively. (For example: In Transcript Excerpt 2 in turn 202, Katie mentions 12 metres, but she means squared metres. She also makes no clear distinction between room dimensions and area. Pedram repeatedly returns to the idea of square rooms. In his comments, the shape of the room and the square unit are confused. But as it was already mentioned the peer communication contributes to Pedram's better understanding of the situation and the final solution of the task.)

5.3 Factors affecting the quality of peer feedback

If we look at feedback as a possible form of assessment, it may be provided (either by the teacher or a classmate) in accordance with the goal of the activity. However, this is difficult when solving an open problem, because its solution usually allows different approaches, that is, the solver can pursue different directions in his problem-solving activity. As we have already stated from the point of view of the long-term goals of education (the competence layer of the problem), the goal of the problem "Fred's Apartment" should be aimed at strengthening the competence of problem-solving. It is difficult for students to provide peer feedback to support the development of this competence.

The thematic layer of the task is underpinned by the task formulation in the form of a word problem, a mathematical problem set within the framework of a realistic situation.[1] Proposals for solutions that had their basis in this layer appeared. However, the quality of the feedback was diverse because the students did not have enough real life experience of different living spaces. We could observe this especially in Transcript 2 and 4. But the most important, from the point of view of mathematics teaching, is the conceptual layer. Here it is necessary to recall that this was an open problem that can be viewed and solved from various points of view. The quality of feedback may increase if the task were focused on a specific mathematical concept. For example, the problem could be directed to the following aims (expressed in the form of teacher's questions):

– What would be a suitable way of sketching the plan (of the apartment)?

– What scale was used for the plan? – ratio in a real situation

– What might be the dimensions of an apartment that is suitable for people to live in?

– What are the relationships between the area of a geometrical figure, its dimensions and shape?

This study contributes to the larger *Social Unit of Learning Project* by examining student group work and their social interactions in terms of peer feedback. The project provides a rich source of classroom data in which researchers could apply different perspectives to interrogate the data. Through this interrogation, researchers could also broaden and deepen their understanding of their own perspectives. By examining the student-student conversation in terms of what feedback the students offer each other in the problem-solving process, this study identifies some of the task and interpersonal conditions that necessitate productive problem-solving processes. Future investigations may involve juxta positioning the different theoretical lenses (for example Brousseau, 1997, Brousseau & Sarrazy, 2002) to identify points of connections and distinctions to enhance the theorisation and analysis of student group work.

Acknowledgment

This research was supported under the Australian Research Council's Discovery Projects funding scheme (Project number DP170102541). We would like to thank the students, parents, teachers, and school staff for their invaluable support of this project.

References

Black, P., & Wiliam, D. (2009). Developing the theory of formative assessment. *Educational Assessment, Evaluation and Accountability*, *21*(1), 5–31. https://doi.org/10.1007/s11092-008-9068-5

Black, P., Harrison, C., Lee, C., Marshall, B., & Wiliam, D. (2004). Working inside the Black Box: Assessment for learning in the classroom. *Phi Delta Kappan*, *86*(1), 8–21. https://doi.org/10.1177/003172170408600105

Boud, D., & Associates. (2010). *Assessment 2020: Seven propositions for assessment reform in higher education.* Australian Learning and Teaching Council.

Boud, D., & Falchikov, N. (2007). *Rethinking assessment in higher education.* Kogan Page.

Brousseau, G., & Sarrazy, B. (2002). *Glossaire de quelques concepts de la théorie des situations didactiques en mathématiques* [English translation V. Warfield]. DAEST, Université Bordeaux 2.

[1]Word problems constitute one of the few school mathematics domains which require mathematization of situations described in words and transformation of the mathematical solution back to the semantic context of the problem. It often happens that the student does not recognize the mathematical model of the problem immediately; one of the tasks given to the student is exactly to discover (or construct) this mathematical model. Word problems also allow development of students' capacity to use their mathematical knowledge in extra-mathematical domains and in this way they contribute to recognition, understanding and memorising of notions, methods and results from mathematics. (Novotná, 2010)

Brousseau, G. (1997). *Theory of Didactical situations in mathematics 1970–1990* [Translation from French M. Cooper, N. Balacheff, R. Sutherland, & V. Warfield]. Kluwer Academic Publishers.

Chan, M. C. E., & Clarke, D. J. (2017). Structured affordances in the use of open-ended tasks to facilitate collaborative problem-solving. *ZDM-The International Journal on Mathematics Education*, *49*(6), 951–963. https://doi.org/10.1007/s11858-017-0876-2

Chan, M. C. E., & Clarke, D. J. (2019). Video-based research in a laboratory classroom. In L. Xu, G. Aranda, W. Widjaja & D. Clarke (Eds.), *Video-based research in education: Cross-disciplinary perspectives* (pp. 107–123). Routledge.

Chan, M. C. E., & Sfard, A. (2020). On learning that could have happened: The same tale in two cities. *Journal of Mathematical Behavior*, *60*, Art. 100815. https://doi.org/10.1016/j.jmathb.2020.100815

Chan, M. C. E., Clarke, D. J., & Cao, Y. (2018). The social essentials of learning: An experimental investigation of collaborative problem-solving and knowledge construction in mathematics classrooms in Australia and China. *Mathematics Education Research Journal*, *30*(1), 39–50. https://doi.org/10.1007/s13394-017-0209-3

Chan, M. C. E., Ochoa, X., & Clarke, D. J. (2020). Multimodal learning analytics in a laboratory classroom. In M. Virvou, E. Alepis, G. A. Tsihrintzis, & L. C. Jain (Eds.), *Machine learning paradigms: Advances in learning analytics* (pp. 131–156). Springer. https://doi.org/10.1007/978-3-030-13743-4_8

Chiu, M. M. (2008). Flowing toward correct contributions during groups' mathematics problem-solving: A statistical discourse analysis. *Journal of the Learning Sciences*, *17*(3), 415–463. https://doi.org/10.1080/10508400802224830

Clarke, D. J. (2001). Untangling uncertainty, negotiation and intersubjectivity. In D. Clarke (Ed.), *Perspectives on practice and meaning in mathematics and science classrooms* (pp. 33–52). Kluwer Academic.

Clarke, D. J., & Chan, M. C. E. (2020). Dialogue and shared cognition: An examination of student–student talk in the negotiation of mathematical meaning during collaborative problem-solving. In N. Mercer, R. Wegerif, & L. Major (Eds.), *International handbook of research on dialogic education* (pp. 581–592). Routledge.

Cohen, E. G. (1994). Restructuring the classroom: Conditions for productive small groups. *Review of Educational Research*, *64*(1), 1–35. https://doi.org/10.2307/1170744

Cohen, E. G., & Lotan, R. A. (2014). *Designing groupwork: Strategies for the heterogeneous classroom* (3rd ed.). Teachers College Press.

Damon, W., & Phelps, E. (1989). Critical distinctions among three approaches to peer education. *International Journal of Educational Research*, *13*(1), 9–19. https://doi.org/10.1016/0883-0355(89)90013-X

Díez-Palomar, J., Chan, M. C. E., Clarke, D., & Padrós, M. (2021). How does dialogical talk promote student learning during small group work? An exploratory study. Learning, Culture and Social Interaction, 30 (Part A), 100540. https://doi.org/10.1016/j.lcsi.2021.100540

Francisco, J. (2013). Learning in collaborative settings: Students building on each other's ideas to promote their mathematical understanding. *Educational Studies in Mathematics*, *82*(3), 417–438. https://doi.org/10.1007/s10649-012-9437-3

Gillies, R. (2016). Cooperative learning: Review of research and practice. *Australian Journal of Teacher Education*, *41*(3), 39–51. https://doi.org/10.14221/ajte.2016v41n3.3 – via eric.ed.gov

Harlen, W., & James, M. (1997). Assessment and learning: Differences and relationships between formative and summative assessment. *Assessment in Education: Principles, Policy and Practice*, *4*(3), 365–379. https://doi.org/10.1080/0969594970040304

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research*, *77*(1), 81–112. https://doi.org/10.3102/003465430298487

Hendl, J. (2005). *Kvalitativní výzkum* [Qualitative research]. Portál.

Hesse, F., Care, E., Buder, J., Sassenberg, K., & Griffin, P. (2015). A framework for teachable collaborative problem-solving skills. In P. Griffin, & E. Care (Eds.), *Assessment and Teaching of 21st Century Skills, Educational Assessment in an Information Age* (pp. 37–56). Springer Science + Business Media. https://doi.org/10.1007/978-94-017-9395-7_2

Isohätälä, J., Näykki, P., Järvelä, S., & Baker, M. J. (2018). Striking a balance: Socio-emotional processes during argumentation in collaborative learning interaction. *Learning, Culture and Social Interaction*, *16*, 1–19. https://doi.org/10.1016/j.lcsi.2017.09.003

Johnson, D. W., Johnson, R. T., & Holubec, E. J. (1994). *The nuts and bolts of collaborative learning.* Interaction Book Company.

Johnson, D. W., & Johnson, R. T. (1984). Structuring groups for collaborative learning. *Organizational Behavior Teaching Review*, *9*(4), 8–17. https://doi.org/10.1177/105256298400900404

Kaur, B., Anthony, G., Ohtani, M., & Clarke, D. (Eds.). (2013). *Student voice in mathematics classrooms around the world.* Sense Publishers. https://doi.org/10.1007/978-94-6209-350-8

Leikin, R., & Zaslavsky, O. (1999). Cooperative learning in Mathematics. *The Mathematics Teacher*, *92*(3), 240–246.

Lohman, M. C., & Finkelstein, M. (2000). Designing groups in problem-based learning to promote problem-solving skill and self-directedness. *Instructional Science*, *28*(4), 291–307.

Moate, J., Kuntze, S., & Chan, M. C. E. (2021). Students' participation in peer interaction – Use of material resources as a key consideration. *LUMAT: International Journal on Math, Science and Technology Education*, *9*(1), 29–55.

Novotná, J. (2010). *Study of solving word problems in teaching of mathematics. From atomic analysis to the analysis of situations.* Germany: LAP LAMBERT Academic Publishing.

OECD. (2005). *Formative assessment: Improving learning in secondary classrooms.* Paris: OECD.

Pellegrino, J. W., Chudowsky, N., & Glaser, R. (Eds.). (2001). *Knowing what students know: The science and design of educational assessment.* National Academy Press.

Ross, J. A., & Smyth, E. (1995). Differentiating cooperative learning to meet the needs of gifted learners: A case for transformational leadership. *Journal for the Education of the Gifted*, *19*(1), 63–82. https://doi.org/10.1177/016235329501900105

Ruiz-Primo, M. A., & Furtak, E. M. (2006). Informal formative assessment and scientific inquiry: Exploring teachers' practices and student learning. Educational Assessment, *11*(3–4), 237–263.

Silver, E. A. (1995). The nature and use of open problems in mathematics education: Mathematical and pedagogical perspectives, Zentralblatt für didaktik der mathematik. *International Reviews on Mathematical Education*, *27*(2), 67–72.

Slavík, J. (2003). *Autonomní a heteronomní pojetí školního hodnocení – aktuální problem pedagogické teorie a praxe* [Autonomous and heteronomous assessment – current issue of educational theory and practice]. *Pedagogika*, *40*(1), 5–25.

Slavík, J., Janík, T., Najvar, P., & Knecht, P. (2017). *Transdisciplinární didaktika: o učitelském sdílení znalostí a zvyšování kvality výuky napříč obory* [Transdisciplinary didactics: about teacher sharing of knowledge and increasing the quality of teaching across disciplines]. Masarykova univerzita, Pedagogická fakulta.

Sluijsmans, D. M. A. (2002). *Student involvement in assessment. The training of peer assessment skills* [Unpublished doctoral dissertation]. Open University of the Netherlands, The Netherlands.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques.* Sage.

Nieminen, J. H., Chan, M. C. E., & Clarke, D. J. (2021). What affordances do open-ended real-life tasks offer for sharing student agency in collaborative problem-solving? *Educational Studies in Mathematics.* https://doi.org/10.1007/s10649-021-10074-9

Tuohilampi, L. (2018). Analyzing engagement in mathematical collaboration: What can we say with confidence? In E. Bergqvist, M. Österholm, C. Granberg & L. Sumpter (Eds.), *Proceedings of the 42nd Annual Meeting of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 235–242). Sweden: PME.

Žantovská, I. (2015). *Rétorika a komunikace* [Rhetoric and communication]. Prague: Dokořán.

Žlábková, I., & Rokos, L. (2013). Pohledy na formativní a sumativní hodnocení žáka v českých publikacích [Formative and summative assessment in Czech publications]. *Pedagogika*, *58*(3), 328–354.