Financial relationships and dependencies in Czech secondary school mathematics textbooks

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This article analyses and compares Czech secondary school mathematics textbooks designated for vocational schools from the perspective of financial relationships and dependencies. Understanding financial relationships and dependencies, such as inflation or the relationship between the potential return of an investment and its riskiness, play an important role in financial literacy and indeed are implicitly present in the Czech Financial Literacy Standard. Functional thinking is an important prerequisite for understanding financial relationships and dependencies. Moreover, financial topics may be motivating and suitable contexts for developing such thinking. We performed a comparative analysis of four sets of Czech secondary mathematics textbooks designated for vocational schools to evaluate the extent to which their tasks may help pupils gain a deeper understanding of financial relationships and dependencies and develop their functional thinking. Our comparative analyses indicate that these mathematics textbooks do not provide teachers and pupils with opportunities to gain insights into the relationships and dependencies in financial education and fail to fully exploit the potential of the tasks with financial contexts to develop pupils' functional thinking. The importance of the study lies in its focus on the connection between financial education and functional thinking, a link which has been missing in the literature we are familiar with.

Key words:
financial literacy, functional thinking, relationships and dependencies, inflation, mathematics textbooks.

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1 Introduction

The current economic situation, accompanied, among other things, by strong inflationary pressure, has led Europe, including the Czech Republic, to search for an optimal way to protect money, which is losing its real value. Individuals are being approached with various investment offers to help them "beat" inflation. However, many do not adequately understand basic financial relationships, including that every investment involves risk, as indicated by the warnings on the Czech National Bank's website (CNB, 23. 5. 2023). In this context, the need to develop individuals' financial literacy as early as possible in their education so that they reach an appropriate level of financial literacy upon leaving school is all the more important. The required level of financial literacy during formal education is defined by the financial literacy standard (MoF, 2017) and is considered to be acquired by the end of secondary school.

The level of financial literacy was measured by a PISA survey in 2012. Participants from the Czech Republic achieved an average result, similar to participants from some other European countries. At the same time, it was found that financial literacy is closely related to mathematical and literacy skills and that the level of financial literacy is influenced by the socio-economic status of the individual or their family (OECD, 2014). In the Czech Republic, this was apparent in the above-average results of pupils from secondary grammar schools (usually attended by individuals from better social and economic backgrounds), in contrast to the average results of pupils from vocational secondary schools and the below-average results of the most socially and economically disadvantaged pupils from vocational schools, who achieved only a basic level of financial literacy (CSI, 2014, 2023). Taking into account the PISA definitions of basic and average levels of financial literacy (OECD, 2014), it can be assumed that pupils with this level of financial literacy, if it were their ultimate level, would have difficulty making responsible financial decisions in their future lives, especially long-term ones.

In our view, the ability to make financial decisions with long-term implications is largely dependent on a deeper understanding of the relationships and dependencies in the world of finance, including the relationship between potential returns and the riskiness of an investment and the dependencies between the inflation rate and the real value of funds. Reasoning in relations and dependencies is an integral part of mathematical literacy, and mathematics education has long been concerned with developing pupils' functional thinking as an intrinsic part of mathematical thinking that enables individuals to think in contexts and notice dependencies between variables. One of the goals of developing functional thinking in pupils is to equip them with the skills to recognise and correctly interpret the various relationships and dependencies they encounter in everyday life (Lichti & Roth, 2019). Personal finance is a good real-life example of an area where pupils should use their previously acquired functional thinking skills.

In the Czech Republic, however, Opletalová (2015) found that teachers often hesitated to teach financial education because they had limited knowledge and skills in this area; this applied to both civics and social studies teachers and mathematics teachers. Even the Czech School Inspectorate (2023), in their

thematical report on financial education, noted that teachers complain of limited methodical support for them concerning financial education. In this context, textbooks become very important as they can support the teacher when covering the not-so-popular topic of financial education. To our knowledge, however, there is currently no research that specifically addresses the content of mathematics textbooks that include financial education topics, much less research that aims to explore the extent to which such textbooks facilitate a deeper understanding of the relationships and dependencies that are essential for responsible financial decision-making.

This article aims to fill this gap, at least partially, by analysing Czech secondary school mathematics textbooks designated for vocational schools to find out to what extent textbooks support teachers in designing teaching focused on a deep understanding of financial relationships and dependencies. Secondary school textbooks are selected as Czech pupils mostly meet financial relationships and dependencies at this stage of their schooling (MoF, 2017).

We first briefly describe the theoretical background of this study and the related literature. We then present the results of the analysis of four sets of secondary school mathematics textbooks, followed by a discussion of our results and conclusion.

2 Theoretical background and literature review

In this section, we first examine the relationship between financial literacy and functional thinking. Next, we present a concept development theory and show how it may be used for task analysis. Finally, we describe what role mathematics textbooks may play in financial education and formulate our research questions.

2.1 Financial relationships and dependencies and functional thinking

In the Czech National Strategy for Financial Education 2.0, financial literacy is defined as "a set of knowledge, skills and attitudes necessary to achieve financial well-being through responsible financial decision-making" (MoF, 2019, p. 4). Similarly, the OECD (2014) defines financial literacy as

> knowledge and understanding of financial concepts and risks and the skills, motivation and confidence to apply such knowledge and understanding in order to make effective decisions across a range of financial contexts, to improve the financial well-being of individuals and society, and to enable participation in economic life. (p. 33)

Both definitions above outline skills in the financial domain and rational decision-making regarding financial problems, whilst the second one adds an understanding of financial concepts. Understanding financial relationships and dependencies is part of the understanding necessary to make effective or responsible financial decisions. Inflation is a classic example. One cannot make a responsible financial decision without a sound understanding of inflation. Nevertheless, all the consequences of inflation can be read as relationships or dependencies, and inflation itself depends on the volume of money in a given economy.

Many authors emphasise the role of pupils' functional thinking in developing their understanding of relationships and dependencies in real contexts. Functional thinking (a term first used by the mathematician F. Klein more than 100 years ago), i.e., the ability to perceive the relationships between phenomena and to reason in terms of cause and effect, can be considered one of the important goals of mathematics education that is applied in the life of every individual (Lichti & Roth, 2019). As stated by Niss (2014):

> functions play crucial parts in the application of mathematics to and modelling of extra-mathematical situations and contexts, e.g., when the development of a biological population is phrased in terms of a nonnegative function of time, when competing coach company tariff schemes are compared by way of their functional representations, or when the best straight line approximating a set of experimental data points is determined by minimising the sum-of-squares function. (p. 239)

There is a two-way relationship between financial education and pupils' functional thinking. Firstly, financial issues provide an attractive context for pupils and students to develop their functional thinking. As Eisenmann and Kopáčková (2006) state, tasks with economic assignments in financial mathematics are well tolerated by pupils in primary and secondary schools; they are intrinsically motivated to solve them. They may, therefore, be good examples (or in terms of the Theory of Generic Models (Hejný, 2012, 2014) suitable isolated models) of the concept of a function and may serve well to gain a deeper understanding of functions in general. Consider that in the financial domain, we may encounter functions that are not continuous and that do not correspond to any commonly studied function in primary or secondary school (such as linear functions, quadratic functions or exponential or logarithmic functions) (Eisenmann & Kopáčková, 2006).

Secondly, thinking more closely about relationships in finance can help pupils develop and deepen their financial literacy. In their future financial decisions, pupils will be confronted with a large amount of data, which may be presented, for example, in the form of graphs or tables, but at the same time, pupils may lack data to make rational and responsible decisions. Taking the simple example of a decision about whether to invest in a corporate bond, the pupil will likely have information about the potential return on the security. To make the right decision, however, the pupil will need to realise that the return on a security depends on its risk (i.e., that a high potential appreciation is a sign of a risky investment and, therefore, less likely to actually achieve a higher appreciation) and that the real appreciation of a security will also be affected by inflation and its expected development over the investment period.

Research on functional thinking has identified several areas where pupils display misconceptions and learning difficulties. In finance, the problem with the interpretation and meaning of functions and problems related to the representation of functions appear to be particularly crucial. Numerous studies have shown that pupils struggle to recognise functions in the real world and everyday experience and cannot interpret them correctly (e.g., O'Shea et al., 2016). Indeed, pupils may not be confronted with such practice-oriented tasks in mathematics textbooks (Mesa, 2004). This also leads them to tend to think of functions mainly in terms of the models they have encountered in mathematics classes (e.g., linear, quadratic, etc.) and thus not identify as functions relationships that do not correspond to these basic mathematical functions or that cannot be expressed simply in a single algebraic expression (Tall & Bakar, 1992). As mentioned above, relationships between variables in finance rarely correspond to simple mathematical functions from mathematics textbooks. An individual with such a limited knowledge of functions from school cannot recognise that they are confronted with a function and cannot reason about the dependent and independent variables, their relationship, etc.

Regarding interpretation, it may also be noted that reasoning about the dependent and independent variables appears problematic. Some pupils do not understand these concepts well (Sierpinska, 1992) and may confuse them, reversing the causal relationship. A typical example from the financial field is the failure to understand that when examining the relationship between potential return and riskiness of an investment, risk is an independent variable, and potential return is a dependent variable. Unlike return, this can lead to underestimating risk, which usually has no specific numerical expression.

Another fundamental problem for understanding the world of finance concerns the representation of functions. Research shows that pupils may have difficulty transitioning between different ways of expressing a function (e.g., graphically, verbally, using tables of values, or algebraically, Dubinsky & Wilson, 2013; Kopáčková, 2005), and even that some difficulties with different representations of functions may persist for teachers (Hitt, 1998). This, again, narrows the understanding of what is and is not a function (Tall & Bakar, 1992). It also makes it impossible for pupils to interpret the relationship presented to them correctly. It is, therefore, necessary to present pupils with sufficient different representations of functions (Sierpinska, 1992). After all, real financial documents also contain data presented and organised in a very different manner (mainly graphs and tables), and understanding a particular relationship between data and the ability to translate from one representation to another is crucial here.

In the next section, we discuss how the Theory of Generic Models can help overcome some difficulties in financial relationships and dependencies noted above.

2.2 Theory of generic models in teaching financial relationships and dependencies

An important premise we are making here is that we can apply theories borrowed from mathematics education to help pupils better understand financial relationships or dependencies, even though this is not how financial relationships and dependencies are usually taught. In mathematics education, concept development theories are particularly important as they allow for the description of pupils' level of understanding but can also provide the basis for designing teaching. In the Czech context, the most widely used theory is the Theory of Generic Models, developed by Hejný (2012). This theory consists of several levels of knowledge acquisition with mental shifts in between. The first is the level of motivation, which should activate pupils and awaken their interest in the subject. This is followed by the level of isolated models, where the pupil obtains enough experience with concrete examples of future knowledge. According to Hejný (2014), it is important to include not only concrete models of such knowledge but also non-models (non-examples of future knowledge), surprising models (models which one would expect not to be a model of future knowledge) and apparent models (they are not models of future knowledge, but may appear to be at first sight). Gradually, the pupil creates a generic model from these isolated models in their mind. The generic model is an example of all the previous isolated models and contains their common elements. The abstract shift then leads to the acquisition of abstract knowledge, which in mathematics is traditionally represented, for example, by algebraic formulas. The whole process is accompanied by what Hejný calls crystallisation, which consists of connecting new knowledge to existing knowledge and any knowledge in the future (Hejný, 2012).

The Theory of Generic models thus describes how pupils acquire new knowledge in mathematics, but it can also be used as a support for designing teaching aimed at deep understanding (and indeed has been, see, e.g., Hejný, 2014). Let us present an example from finance. Suppose we teach about the phenomenon of inflation. After motivation, the pupil should gradually be confronted with isolated models, that is, various manifestations of inflation, i.e., the increase in price levels (or simply price increases), but also the loss of the value of money over time. This is essential if the pupil is to realise later that inflation impacts not only prices but also funds deposited in the bank, funds in savings or investment products, etc. Such a realisation would be a generic model later abstracted into the formula for calculating inflation or its economic and statistical definition. When such knowledge is further crystallised, one can imagine that the pupil can realise, for example, why more loans or increased pensions create inflationary pressures. The teaching designed according to the stages of the Theory of Generic Models aims to reach a deep understanding of concepts and relationships in pupils.

In the next section, we discuss the role of textbooks in teaching financial education and how the previously mentioned theories and findings may be translated into them.

2.3 Role of textbooks

In teaching financial education, as well as in teaching other topics, textbooks play a major role. Teachers perceive textbooks as a curriculum translation (Chiappetta & Fillman, 2007). Textbooks influence their teaching of mathematics, what and how pupils learn, and what experience pupils take away from the lessons (Tarr et al., 2006). As Johansson (2006) demonstrated, textbooks may be used as a teacher's guide for organising mathematics lessons, including the motivational part of the lessons.

Taking into account the wide use of textbooks in mathematics lessons (Mullis et al., 2012) and a certain degree of fear of teaching financial education or self-reported lack of teaching support among Czech teachers (Opletalová, 2015; CSI, 2023), it is highly likely that textbooks and their content would also play a crucial role in teaching financial literacy in mathematics and in shaping the experience pupils take away from their lessons on financial education in mathematics.

Considering the above, we may ask to what extent textbooks support the design of teaching aimed at deep understanding. For example, Pang and Sunwoo (2022) analysed a Korean elementary mathematics textbook related to functional thinking based on the premise that specific tasks may foster functional thinking. Shield and Dole (2012) used, among others, theories of learning to explore the potential of textbooks to promote deep learning. Zenkl (2021) analysed Czech mathematics textbooks for secondary schools from the perspective of the Theory of Generic Models while investigating combinatorial concepts aimed at deep understanding.

In the context of chapters on financial education, the presence of particular isolated models in the textbooks (e.g., non-models, surprising models or apparent models) would help pupils to gain a deep understanding of financial relationships and dependencies. According to Hejný (2012), those specific models would help pupils to build their generic model and to prevent them from gaining mechanical knowledge. Likewise, some tasks presented to pupils may offer them an opportunity to look for common features and generalise their previous knowledge (and so offer them an opportunity to create a generic model).

Furthermore, as summarised by Pang and Sunwoo (2022) in relation to the role of mathematics textbooks in fostering functional thinking, it may be promoted by using real-life contexts in tasks, using various pattern tasks (numeric and geometric patterns, additive and multiplicate relationships). As also stated by the same authors, functional thinking may be fostered by using tasks that lead pupils to explore relationships by looking at the change in two quantities and using tasks that lead pupils to represent relationships symbolically.

Tasks related to the financial world are, in principle, very close to the real life of pupils. As stated above, financial documents contain much data in table and graph form, which can be analysed in terms of dependent and independent variables, during which we always need to simultaneously analyse changes in both dependent and independent variables if we want to understand a particular relationship correctly (e.g., a change in the potential return of an investment offer accompanied by a change in its riskiness, or a change in the return of a saving product in response to a change in its liquidity, etc.). Thus, there is an opportunity to foster pupils' functional thinking if confronted with well-designed mathematical tasks relating to financial relationships and dependencies in their mathematics textbooks.

2.4 Research questions

Based on the literature review above, the following research questions are formulated:

RQ1: What financial relationships and dependencies from the Financial Literacy Standard currently in force in the Czech Republic can be found in the selected textbooks?

RQ2: Which topics in the field of financial education are presented in more detail, and which are missing in the textbooks? What types of financial relationships and dependencies are included?

RQ3: Do tasks with a financial context in the textbooks allow pupils to build quality generic models of financial relationships and dependencies?

RQ4: Do tasks with a financial context in the textbooks facilitate pupils' functional thinking?

3 Methodology

In this section, data selection and analytical methods are discussed.

3.1 Data

We analysed the Czech Financial Literacy Standard (2017) and four sets of mathematics textbooks used in Czech secondary schools. These are sets of textbooks commonly available and used in schools. They are approved for teaching by the Ministry of Education, Youth and Sports of the Czech Republic.

Textbook sets that are designed exclusively for secondary grammar schools are not included in the sample, as this study does not focus on this specific segment of relatively prestigious secondary schools, bearing in mind, like Dubinsky and Wilson (2013), that relatively little research has focused on the learning needs of rather below-average pupils in relation to specific mathematical concepts. Thus, we analysed the textbooks which, according to their authors, can be used in different types of secondary schools (Tab. 1).

Tab. 1: Textbooks included in the study

Publisher	Textbook series name	Volumes included in the study	Authors	Year of publication	Code designation of the set in the study
Didactics	Matematika pro střední odborná učiliště [Mathematics for vocational schools]	Volume 1: Numbers and expressions	Marková, Siebenbürgerová, Zemek, & Macálková	2020	D
		Volume 2: Equalities, Inequalities, Functions	Marková & Macálková	2021	
		Volume 4: Stereometry, Work with Data	Květoňová, Marková, & Macálková	2021	
Prometheus	Matematika pro netechnické obory SOŠ a SOU [Mathematics for non-technical programmes of secondary and vocational school]	Volume 1	Calda	1996	P1
		Volume 2	Calda	1997	
		Volume 3	Calda	1998	
Prometheus	Matematika pro střední odborné školy a studijní obory středních odborných učilišť [Mathematics for secondary schools and selected programmes of vocational schools]	Volume 1	Calda, Petránek, & Řepová	1996	P2
		Volume 2	Odvárko, Řepová, & Skříček	2001	
		Volume 3	Odvárko & Řepová	2009	
		Volume 4	Petranek, Calda, & Hebák	2002	
		Progressions and Financial Mathematics	Odvárko	2002	
Fraus	Matematika s nadhledem od prváku k maturitě [Mathematics with an overview from freshman to high school diploma]	Volume 1: Number Fields, Sets	Fuchs & Tlustý	2019	F
		Volume 2: Algebraic Expressions	Fuchs & Koldová	2019	
		Volume 3: Equalities and Inequalities I	Zhouf	2019	
		Volume 4: Equalities and Inequalities II	Zhouf	2019	
		Volume 5: Functions	Tlustý	2019	
		Volume 9: Goniometry and Trigonometry	Tlustý & Pomykalová	2019	
		Volume 13: Combinatorics, Probabilities, Data	Tlustý	2020	
		Volume 14: Progressions	Tlustý	2020	

All the textbooks included in the study have a similar structure. Each chapter consists of an explanation of the new subject matter, sample tasks, and tasks to be solved by pupils.

The parts that deal with financial mathematics are studied from each set of textbooks. Tab. 2 presents the number of pages of such parts and shows that chapters on financial mathematics usually form a negligible part. This chapter represents less than 1% of the content in two textbooks. For the other two textbooks, the proportion is slightly higher.

Tab. 2: Size of the chapters on financial mathematics in the textbooks

Set of textbooks	Number of pages in the chapter on financial mathematics	Number of pages in the whole set of textbooks	Percentage of the textbook set devoted to the chapter on financial mathematics
D	6	408	1.5%
P1*	7	865	0.8%
P2**	45	973	4.6%
F	10	1 201	0.8%

* This set of textbooks does not contain any chapter dealing directly with financial mathematics, but this topic is mostly represented in the chapter on the application of progressions.
** This set of textbooks contains only a chapter on using progressions in financial mathematics.

Next, we analyse the parts of the textbooks that might contain tasks with a financial context that potentially target pupils' functional thinking (in particular, the parts that focus on numbers and operations on numbers, expressions, equations and inequalities, functions, statistics and probability). We did not analyse any of the parts or chapters in the textbooks on plane geometry, solid geometry, or analytical geometry.

The textbooks in set D also include integrated workbooks, but they are not examined as the other textbooks have separate workbooks, and we wanted to keep the same conditions of comparison for all four sets of textbooks. Similarly, all the workbooks are prepared by the same authors as the textbooks. Thus, the type of tasks presented in the workbooks might be very similar to those in the textbooks, and further analysis of all the workbooks would not, in our opinion, change the results of this analysis significantly.

3.2 Data analysis

Following Fan et al.'s (2013) recommendations, the textbooks are analysed according to predefined criteria and then compared based on these criteria. The predefined criteria, summarised in Tab. 3 below, are based on our research questions and previous literature overview.

First, we performed a qualitative content analysis of the current Czech Financial Literacy Standard (2017) to determine the targeted financial relationships and dependencies. They became part of the analytical framework and are listed in Appendix I.

Secondly, based on the Financial Literacy Standard (2017) analysis, we analysed the chapters on financial mathematics in four textbooks. In these chapters, we examined the context of word problems in relation to the Financial Literacy Standard applicable in the Czech Republic (individual topics are listed below in Tab. 4). If the context for a task was related to the financial relationships and dependencies presented in Appendix I, we further investigated whether the task could contribute to developing a generic model of a given relationship and to developing pupils' functional thinking. For example, if we found a task asking for an interpretation of a graph representing the relationship between the inflation rate and the real value of savings, we would consider such a task as developing pupils' functional thinking and also helping them to develop a generic model of inflation (such as the phenomenon of depreciation of the real value of money).

Then, similarly, in the textbook sets studied, we considered other chapters potentially including tasks with a financial context which could be used to develop pupils' functional thinking and analysed tasks included in those chapters based on the same criteria.

4 Results

In this section, we present the main descriptive results of the analysis.

4.1 Financial relationships and dependencies in the current Czech Financial Literacy Standard (RQ1)

Appendix I presents which dependencies appear in the Czech Financial

Literacy Standard outputs. However, some are not explicitly named as such. For example, the word "influence" is used instead. In Appendix I, for each output of the Standard, we add what kind of relationship is targeted and what its basis is.

Tab. 3: Criteria for the textbook analysis

Research question	Short description	Criteria of analysis			
		Criterion 1	Criterion 2	Criterion 3	Criterion 4
RQ1	*Financial relationships and dependencies in the Czech Financial Literacy Standard*	Expected output in the Financial Literacy Standard of financial education relates to understanding the relationship between two variables (two financial phenomena)			
RQ2	*Financial topics and relationships and dependencies in textbook tasks*	A task has a financial context, so it relates to at least one topic of the Czech Financial Literacy Standard (2017)	A task deals with at least one of the financial relationships and dependencies from the Czech Financial Literacy Standard (2017)		
RQ3*	*Building up a generic model of a financial relationship or dependency*	There is a sufficient number of isolated models of a given financial relationship in textbooks.	A task presents at least one of the following models of a given financial relationship to pupils: non-model, surprising model, apparent model.	One or more tasks ask pupils to look for common features of presented isolated models.	
RQ4**	*Development of functional thinking*	Pupils are asked, based on a given graph, table or expression relating to a given financial relationship, to: – interpret a given relationship verbally – determine dependent and independent variable – estimate missing values	Pupils are asked, based on a given verbal description or table of values of a given financial relationship, to express it: – symbolically (as an expression) – graphically	Pupils are asked in a task relating to finance to work with an unusual functional representation (e.g., non-linear function, discontinuous function)	A task relating to finance has a strong motivational potential because it relates to real-life context, and the financial relationship in the task may be new or interesting for pupils

* The more criteria met, the higher the likelihood that pupils will create quality generic models in their minds. A sufficient number of isolated models cannot be exactly quantified, but the premise is that the more isolated models, the higher the likelihood of quality generic models.
** To conclude that a given task helps pupils foster their functional thinking, one of criteria 1,2 or 3 must be met. Criterion 4 is complimentary and relates to the motivational potential of those tasks.

In general, it can be summarised that two types of relationships or dependencies are implicit in the outcomes of the Financial Literacy Standard. One type relates to inflation and its causes and effects on financial resources in various forms. The second type concerns the relationships and dependencies between different characteristics of financial products (e.g., relationships between riskiness and potential return on investment, between liquidity and appreciation of funds held in saving products). Appendix I is provided to this article for more detailed information relating to this research question.

In the next section, we will answer whether these two types of relationships are included in the analysed textbooks.

Tab. 4: Financial topics covered by tasks in the financial mathematics chapters*

Textbook	Topic	Shopping and paying			Economy	Household budget surplus				Household budget deficit	
	Subtopic	shopping	paying	inflation	budget, income, expenses	savings	investments	insurance	security for old age	interests and loans	non-repayment
Set D	Number of tasks	4	4	0	8	3	0	0	0	3	0
	Total number of tasks	22	22	22	22	22	22	22	22	22	22
	%	18.2	18.2	0	36.4	13.6	0	0	0	13.6	0
Set P1	Number of tasks	0	0	0	0	7	0	0	0	1	0
	Total number of tasks**	8	8	8	8	8	8	8	8	8	8
	%	0	0	0	0	87.5	0	0	0	12.5	0
Set P2	Number of tasks	0	0	0	0	54	6	0	0	31	0
	Total number of tasks	91	91	91	91	91	91	91	91	91	91
	%	0	0	0	0	59.3	6.6	0	0	34.1	0
Set F	Number of tasks	0.5	0	2	0	8.5	0	0	0	4	0
	Total number of tasks	15	15	15	15	15	15	15	15	15	15
	%	3.3	0	13.3	0	56.7	0	0	0	26.7	0

* The table includes both sample tasks and tasks used to practise what is learned.
** This value represents the total number of tasks in financial mathematics in a broader chapter on the use of geometric progressions.

4.2 Financial topics in Czech secondary school textbooks (RQ2)

In each textbook, word problems are used in the chapter on financial mathematics. The areas of financial mathematics covered by these tasks are listed in Tab. 4.

As the table shows, some topics are not represented (e.g., insurance or consequences of non-repayment), and others are only exceptional. Only set P2 deals specifically with the issue of investments and only in 6.3% of the tasks. Only set F deals with inflation and only in 13.3% of the tasks (there are two tasks, one sample and one for pupils to practise). The dominant topic of the chapters devoted to financial mathematics in the P1, P2 and F textbooks is savings, which is dealt with in most tasks (56.7% to 87.5%). In set D only, the dominant topic is a budget and its income and expenses (especially calculations related to wages, taxes and levies), representing 36.4% of the tasks in the chapter. The second most represented topic in the P1, P2 and F textbooks is loans (interest rate, repayment calculations, etc.), covered in 12.5% to 32.6% of the tasks in the chapter. In set D alone, the subtopics of paying and shopping are more represented (18.2% each). In comparison, the issue of loans is represented in only 13.6% of the tasks (which is, however, a difference of only one task compared to shopping and paying, but this topic is still represented more than in the P1 textbook, which devotes only one task to it, representing 12.5%). The analysis also shows that only set D covers all the topics from the financial literacy standard in the chapter on financial literacy, and overall, it covers the most subtopics (5 in total) of all the textbooks analysed. Also, it cannot be said that any topic is dominant in set D (all topics have task coverage ranging from 13.6% to 36.4% of the tasks in the chapter).

The use of the financial context for the tasks in the chapters not directly dealing with financial mathematics is shown in Tab. 5.

Setting aside textbook set D, the financial context is used only anecdotally in tasks outside the financial mathematics chapters. In sets P1, P2 and F, it is identified in fewer than ten tasks, representing about 1% of all tasks examined each time. Only in textbook set D did tasks with a financial context occur in 12.5% of the tasks examined. In terms of specific financial topics covered in other subject areas of the textbooks, the topic of shopping and paying (especially calculating prices, discounts, etc.) is present in all textbooks. In sets D, P2 and F, the topic of the economy (of an individual, group or firm, i.e., calculations of take-home pay, levies, taxes, costs, etc.) also appeared to a lesser extent, and in set F, the issue of savings is also present. In sets P1 and F, tasks occur most frequently in the chapters devoted to numerical fields and expressions. In contrast, in sets D and P2, tasks occur most frequently in the chapters devoted to statistics and work with data.

Tab. 5: Financial context in tasks in other textbook chapters*

Textbook	Thematic unit	Equations, inequations, their systems	Functions	Statistics and data	Combinatorics and probability	Number fields, percentages, expressions	Arithmetic progressions**	Total
Set D	Number of tasks	18	0	19	4	34	unable to determine**	75
	Total number of tasks	137	120	58	37	248		600
	%	13.1	0.0	32.8	10.8	13.7		12.5
Set P1	Number of tasks	3	0	0	0	4	1	8
	Total number of tasks	152	201	58	87	99	34	631
	%	2.0	0.0	0.0	0.0	4.0	2.9	1.3
Set P2	Number of tasks	2	3	3	0	0	1	9
	Total number of tasks	68	275	40	128	154	79	744
	%	2.9	1.1	7.5	0.0	0.0	1.3	1.2
Set F	Number of tasks	2	1	0	0	3	0	6
	Total number of tasks	195	281	26	106	184	85	877
	%	1.0	0.4	0.0	0.0	1.6	0.0	0.7

* The table includes both sample tasks and tasks used to practise the material. Their numbers in the table result from adding up the number of tasks related to the topic in different chapters (e.g., the numbers of tasks from the chapters on linear equations and inequalities and their systems and quadratic equations and inequalities are grouped in the table under the thematic unit of equations, inequalities, their systems).
** Progressions are not part of the vocational school curriculum and are therefore not represented in the set D textbooks.

4.3 Tasks to build a generic model of financial relationships and dependencies (RQ3)

As section 4.2 shows, inflation is covered only marginally in the textbooks studied. Only set F contains two tasks dealing with inflation (one sample task and one task to be solved). In both cases, however, the primary aim is to show, following the title of the respective chapter, the use of geometric progressions in financial mathematics. The task asks pupils to calculate the real value of CZK 1,000,000 after one year and ten years at a specified inflation rate of 3.2% (Fig. 1). The practice task asks in how many years prices will double if inflation is a%. Pupils are asked to solve the task in general and then for an inflation rate of 3% (Fig. 2). None of the other tasks in the chapter on loans or savings take inflation into account.

In set F, the chapter on statistics shows the evolution of the inflation rate in the Czech Republic and demonstrates using a line graph to capture the trend over time. However, this is only an example in the theoretical part of the chapter demonstrating how to create a line chart from table data in an Excel table. It is, therefore, not captured in Tab. 5. At the same time, such an example does not provide a deeper insight into inflation put forward by the financial literacy standard for secondary school.

Section 4.2 shows that the main financial products mentioned in the textbooks are savings products and less so loan products. Only in P2 are there investment products, albeit very limited. However, all the tasks in all sets of textbooks are mainly focused on calculations, which demonstrate the use of geometric progressions or calculations of percentages (only the percentage calculation is applied in set D, as progressions are not included in the content of the framework curricula for secondary vocational schools). None of the tasks focusing on savings products includes inflation, nor do any of the tasks look at the relationship between the appreciation of savings through different savings products and their liquidity. Similarly, none of the tasks related to loan products takes into account the effects of inflation on these products (e.g., the effect that inflation may have on the interest rates offered, the fact that inflation reduces the real value of annuity payments) or the extent to which different characteristics of the borrower may affect the interest rate provided. In set P2, the potential appreciation of investment products is again calculated without considering the relationship between this potential appreciation and the riskiness of the instruments.

4.4 Developing functional thinking in tasks with a financial context (RQ4)

The chapters on financial mathematics in all the textbooks studied lack any representation of a function or dependence using a graph, table, word problem or expression.

Task 1
Calculate how much CZK 1,000,000 will be worth a) in one year if the annual inflation rate is 3.2%, b) in 10 years if the average annual inflation rate is 3.2%.

solution

a) in one year

Step 1

We will use the formula $K = K_0 \cdot \left(\frac{1}{1+r}\right)^n$.

Step 2

In this case, $K_0 = 1{,}000{,}000$, $r = 0.032$, $n = 1$.

After substitution, we get: $K = K_0 \cdot \left(\frac{1}{1+r}\right)^n = 1{,}000{,}000 \cdot \frac{1}{1.032} \doteq 968{,}992$

conclusion: Due to inflation, the value drops to CZK 968,992, i.e., we lose about CZK 31,000 out of a million in a year.

b) in 10 years

Step 1

Again, we start from the formula $K = K_0 \cdot \left(\frac{1}{1+r}\right)^n$.

Step 2

In this case, we substitute $K_0 =$ CZK 1,000,000, $r = 0.032$, $n = 10$.

After substitution, we get: $K = K_0 \cdot \left(\frac{1}{1+r}\right)^n = 1{,}000{,}000 \cdot \left(\frac{1}{1.032}\right)^{10} \doteq 729{,}799$

conclusion: Due to inflation, the value will drop to CZK 729,799, i.e., we will lose about CZK 270,000 from the million over ten years.

Fig. 1: Sample task on inflation in set F (volume 14, p. 47); the original Czech version is in Appendix II

Task 6
Annual inflation is always a%. In how many years will prices double? Solve in general and then for $a = 3$%.

Fig. 2: Task on inflation to be solved in set F (volume 14, p. 52); the original Czech version is in Appendix II

The financial context is also little used in the chapters devoted to functions, as shown in Tab. 5. In the case of sets D and P1, the financial context is not used at all in the chapters devoted to functions. In contrast, it is used only minimally in the case of the P2 and F sets.

There is a limited number of tasks in the P2 set of textbooks showing how the final price of the goods or services purchased depends on the number of units purchased. The tasks either require the determination of a function expression based on a word problem or the construction of a graph of the function. These few tasks can help to develop pupils' functional thinking. For example, they include a function that is not continuous (it is a task capturing the relationship between the prices of postal parcels and their size – see Fig. 3), a useful isolated model of a function that may not match pupils' ideas about functions.

2. The charge for a parcel mailed at the post office was previously determined by weight as follows:

up to 20 g	8 CZK
over 20 g to 50 g	10 CZK
over 50 g to 200 g	12 CZK
over 200 g to 350 g	14 CZK
over 350 g to 500 g	16 CZK
over 500 g up to 1 kg	20 CZK

Determine the function that expressed the relevant dependence (i.e. the dependence of the amount of the charge on the numerical value of the parcel weight) and construct its graph.

Fig. 3: Task from textbook set P2 (part 3, p. 20); the original Czech version is in Appendix II

In Set F, the financial context is used in one task related to savings in the section on logarithmic functions (Fig. 4). However, it is an example task. Hence, the pupils receive the solution from the textbook's authors, who marked the task as a task of higher difficulty.

Example 9
Determine after how many years the amount in the account will be more than CZK 1,000,000 if CZK 500,000 is deposited at 4% per annum compound interest (for simplicity, do not consider tax on interest).

solution

Step 1
Compound interest means that the bank credits the depositor with interest at the end of each interest period, and interest is calculated on the amount accrued in the following period.

Step 2
At time $t = 0$ (at the beginning), we have CZK 500,000 deposited.

Step 3
At time $t = 1$ (at the beginning of the second year), we have deposited $500{,}000 \cdot 1.04 =$ CZK520,000.

Step 4
At time $t = 2$ (at the beginning of the third year), we have $500{,}000 \cdot 1.04^2 =$ CZK 540,800.

Step 5
Denote by x the number of interest periods (years) we want to find. We get the inequation:
$500{,}000 \cdot 1.04^x \geq 1{,}000{,}000$

Step 6
Adjust to the form:
$1.04^x \geq 2$

Step 7
Take the logarithm of the two sides of the inequality with the base 10 logarithm. Since the base 10 logarithm is an increasing function, the inequality is preserved, and we get:
$x \cdot \log 1.04 \geq \log 2$
$x \geq \frac{\log 2}{\log 1.04} \doteq 17.67$

conclusion
After 18 years, there will be more than CZK 1,000,000 in the account.

Fig. 4: Sample task from textbook set F (part 5, p. 75); the original Czech version is in Appendix II

5 Discussion

5.1 Financial relationships and dependencies in the Czech Financial Literacy Standard (RQ1)

As stated above, the Financial Literacy Standard (2017) does not explicitly mention financial relationships or dependencies. However, our analysis demonstrates that two types of financial relationships and dependencies are included in the Standard. The first consists of relationships related to inflation and its causes and effects, and the second includes relationships existing among different characteristics of financial products. In our opinion, these financial relationships and dependencies are crucial for rational financial decision-making, and it is natural that the Standard contains them. However, it is questionable to what extent Czech teachers understand these relationships and dependencies, given their general fear of financial education and lack of a deep understanding of financial concepts (Opletalová, 2015; CSI, 2023). For this reason, the role of textbooks and their content is very important.

5.2 Space devoted to the financial relationships and dependencies in the textbooks and the potential of tasks to build a generic model of financial relationships and dependencies (RQ2 and RQ3)

Our study demonstrates that only limited space is devoted to financial relationships and dependencies related to inflation and financial products in the Czech secondary school mathematics textbooks. The main topics covered by the chapters on financial mathematics are savings and loans in three of four textbooks under analysis (P1, P2, F). However, the role of the tasks in the chapters is mainly to demonstrate how to use geometric progression rather than to make clear the relationships in question. The same applies to the two tasks related to inflation in set F. Furthermore, a brief explanation of inflation comes before the tasks. Thus, it is clear that inflation is used in set F as one of the isolated models of geometric progressions or as a simple illustration of calculation with a formula $K = K_0 \cdot [1 : (1 + r)]^n$ rather than a separate financial relationship to be studied. Only textbook set D emphasises individual budgets, including wages, expenses, and shopping and paying, but no space is devoted to inflation or the characteristics of financial

products. We find no task devoted to the representation of relationships and dependencies existing among characteristics of financial products in any of the textbooks. No task introduces any non-model, apparent model or surprising model (in terms of Hejný, 2014) of relationships relating to inflation or characteristics of financial products.

Thus, it can be concluded that none of the textbooks analysed provides pupils with enough opportunities (isolated models, Hejný, 2012) to make a quality generic model of financial relationships, thus reaching the implicit goal in the Czech Financial Literacy Standard (2017). It may also indicate that some professionals (at least the textbooks' authors and curriculum makers) share the belief that financial education should not be part of mathematics education and should not be given more space in mathematics classes at the expense of other, more traditional mathematical topics (Najvar, 2014).

More broadly, the question is to what extent textbooks on financial education are influenced by the attitudes of their authors, or society as a whole, towards financial issues, and to what extent these attitudes are transferred to the training of teachers and pupils' minds. Our study shows that the dominant topics are savings, price issues (but usually without inflation), and less so loan issues, and virtually no investment or insurance products. This may mean that a financially literate individual should, above all, save, borrow little (and judiciously) and pay attention to prices (though, somewhat paradoxically, not inflation). In the case of set D, which is intended only for secondary vocational schools whose graduates, often from low socio-economic backgrounds, are expected to work mainly in professions requiring manual labour, the dominant theme is household management, securing income and paying various (especially mandatory) expenses. It may be a deliberate intention to teach these pupils mainly to manage their own resources without considering that these pupils will ever have greater opportunities to work with a household budget surplus. The financial literacy standard addresses attitudes in financial education only marginally, mentioning, for example, that pupils live in proportion to their financial capabilities and build financial reserves but also that pupils strive to achieve financial well-being. The content of mathematics textbooks may, therefore, contradict at least some of the expected attitudes mentioned in the Financial Literacy Standard (2017).

5.3 Development of functional thinking in the textbooks and financial context (RQ4)

Many researchers highlight the importance of using different representations of functions (graphs, tables, word problems, etc.) to develop pupils' functional thinking and recommend asking pupils and students to interpret these different representations (Dubinsky & Wilson, 2013; Kopáčková, 2005; Sierpinska, 1992). Furthermore, some researchers (e.g., Eisenmann & Kopáčková, 2006) recommend using a financial context in tasks to develop pupils' functional thinking because of its motivational potential.

Our analyses show that only sets P1 and F used a financial context and only in the limited number of functions-related tasks. Such tasks in set P1 could potentially develop pupils' functional thinking. For example, authors in the P1 set work with a function that is not continuous (in a task capturing the relationship between the prices of postal parcels and their size – see Figure 3), which is a useful isolated model of a function that may not match the ideas that pupils have about functions (Eisenmann & Kopáčková, 2006). It also makes pupils switch between different ways of representing functions (e.g., graphs and algebraic expressions, descriptions in words or using a table, etc.), which is not easy for them (e.g., Hitt, 1998). On the other hand, the financial context used in these tasks is somewhat trivial for tasks intended for secondary school pupils, who can be assumed to know that the price of a good or service is based on the number of units, acts, etc. It is noteworthy that price determination is only mentioned in the primary school section, not the secondary one in the Financial Literacy Standard (MoF, 2017). Thus, the question arises whether such tasks fulfil their motivational potential for secondary pupils, as discussed by Eisenmann and Kopáčková (2006) or whether they are too trivial for them.

In Set F, the financial context is combined with the logarithmic function (see Fig. 4). Still, the essence of the task lies in the application of numerical operations and solving inequalities, not in the conceptual understanding of financial relationships. Therefore, whether this task could develop pupils' functional thinking is questionable.

6 Conclusion, limitations and further research

The importance of the study lies in its focus on the connection between financial education and functional thinking, a link which has been missing in the literature we are familiar with. Functional thinking is an important prerequisite for understanding financial relationships and dependencies, whilst financial topics may also be motivating and suitable contexts for developing functional thinking.

The results above indicate that Czech secondary mathematics textbooks do not provide teachers and pupils with opportunities to gain insights into relationships and dependencies in financial education. This could be problematic because a deeper understanding of financial relationships and dependencies is essential for rational financial decisions. If one is unaware of the impact of inflation on savings or investment products, one will hardly be able to choose appropriate products. If one does not understand the relationship between the return and risk of an investment, one can easily fall prey to misleading behaviour by various financial advisors or outright fraudsters. If one does not know how to diversify one's portfolio to mitigate the risk of its devaluation, one cannot manage the available funds rationally.

The study also has some limitations. First, some problems with a financial context might be included in the chapters not analysed in our study (e.g., chapters on plane geometry or solid geometry), even though such cases would probably be only rare, considering how few problems are devoted to the financial context in chapters where we would expect them. Similarly, we did not investigate to what extent this issue is addressed in secondary civics and social studies textbooks and how they didactically grasp the issue, given that many financial literacy topics are included in the social studies education in the framework curricula. This task remains for future research. Third, we discovered that the analysed textbooks do not provide opportunities for conceptual understanding of financial relationships. Nevertheless, we cannot say how teachers use them in lessons or whether they augment them with their own materials to fill the gaps. This also opens a possible new line of research. Finally, we focus on the tasks alone, without considering how teachers would implement them in the lessons and how pupils would solve them. Thus, we can only speak about tasks potentially promoting (or not) deep learning. The study of the use of tasks in real classrooms is a possible follow-up study.

More broadly, the question is to what extent mathematics textbooks and their components on financial education are influenced by the attitudes of their authors, or society as a whole, towards financial issues, and to what extent these attitudes are transferred to the training of teachers and pupils' minds. Our study shows that some of the topics of the Financial Literacy Standard (2017) are not covered by the textbooks at all or only in a very limited and selective way. In this respect, it would be interesting to examine them more closely from the perspective of the theory of the hidden curriculum, which deals with those aspects of education that are not formalised anywhere yet can have a major impact on pupils' value orientations and attitudes (Jackson, 1968).

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Appendix I: Financial Literacy Standard (2017) outcomes assuming mastery of relationships and dependencies

Thematic area of financial education	Education level	Topic	Output according to Standard	The relationship, the understanding of which is assumed by the output
Shopping and paying	lower secondary school	shopping	[pupil] describes the effect of inflation on the value of money	the relationship between the inflation rate and the real value (the higher the inflation rate, the lower the real value of money)
	s upper secondary school	Inflation	[pupil] explains the effect of inflation on income, deposits, loans	the relationship between the inflation rate and real incomes the relationship between the inflation rate and the real appreciation of funds in savings products or current bank accounts the relationship between the inflation rate and the interest rate on a loan the relationship between the inflation rate and the real value of repayments
		inflation	[pupil] suggests how a citizen can protect themselves against moderate and high inflation	the relationship between the amount of money in the economy and inflation (inflation as a function of the amount of money in the economy)
Household economy	lower secondary school	introduction to financial services	compares financial products, particularly in terms of risk, return and liquidity	the relationship between the potential return on an investment and its potential riskiness the relationship between the potential appreciation of the funds saved and its liquidity
		planning	identifies risks to the achievement of financial goals and proposes ways to mitigate them	the relationship between the inflation rate and the real value of savings the relationship between the inflation rate and the real appreciation of the investment product
	upper secondary school		to deepen the level of mastery	all previous
Household budget surplus	upper secondary school	savings	[pupil] selects the appropriate savings product	the relationship between the inflation rate and the real value of savings the relationship between the potential appreciation of savings and liquidity
			[pupil] calculates how long to save for a certain purpose	the relationship between the inflation rate and the real value of savings
		investment	[pupil] assesses different types of investments, including investments in property	the relationship between the potential return on an investment and its potential riskiness the relationship between the inflation rate and the real appreciation of an investment

			[pupil] uses an example to show the necessity of spreading the risk	the relationship between the number of different investment instruments held in the portfolio and the risk of impairment of the investment portfolio
		security for old age	[pupil] describes the options for security in old age	the relationship between the inflation rate and the real appreciation of savings
Household budget deficit	upper secondary school	interests, loans	[pupil] explains the methods of determining interest rates and the use of APRC	the relationship between the inflation rate and the level of interest rates the relationship between various variables (characteristics of the borrower and the loan itself) and the level of the interest rate
			[pupil] selects the most suitable loan in the example	see above
			[pupil] estimates or finds the normal interest rate and APRC of common types of loans	see above

Appendix II: Original versions of tasks

Příklad 1

Vypočtěte, jakou hodnotu bude mít 1 000 000 Kč a) za rok, pokud bude roční inflace ve výši 3,2 %, b) za 10 let, pokud bude průměrná roční inflace ve výši 3,2 %.

řešení

a) za rok

1. krok

Využijeme vzorec $K = K_0 \cdot \left(\frac{1}{1+r}\right)^n$.

2. krok

V tomto případě je $K_0 = 1\,000\,000$, $r = 0{,}032$, $n = 1$.

Po dosazení dostáváme: $K = K_0 \cdot \frac{1}{1+r} = 1\,000\,000 \cdot \frac{1}{1{,}032} \doteq 968\,992$

závěr: Vlivem inflace klesne hodnota na 968 992 Kč, tj. během roku ztratíme z miliónu asi 31 000 Kč.

b) za 10 let

1. krok

Opět vyjdeme ze vzorce $K = K_0 \cdot \left(\frac{1}{1+r}\right)^n$.

2. krok

V tomto případě dosadíme za $K_0 = 1\,000\,000$ Kč, $r = 0{,}032$, $n = 10$.

Po dosazení dostáváme: $K = K_0 \cdot \left(\frac{1}{1+r}\right)^n = 1\,000\,000 \cdot \left(\frac{1}{1{,}032}\right)^{10} \doteq 729\,799$

závěr: Vlivem inflace klesne hodnota na 729 799 Kč, tj. během 10 let ztratíme z miliónu asi 270 000 Kč.

Fig. 5: Task in Figure 1

Cvičení 6

Roční inflace je vždy a %. Za kolik let se ceny zdvojnásobí? Řešte obecně a pak pro $a = 3$ %.

Fig. 6: Task in Figure 2

2. Poplatek za psaní podané na poště byl dříve určován podle hmotnosti takto:

do 20 g	8 Kč
přes 20 g do 50 g	10 Kč
přes 50 g do 200 g	12 Kč
přes 200 g do 350 g	14 Kč
přes 350 g do 500 g	16 Kč
přes 500 g do 1 kg	20 Kč

Určete funkci, která vyjadřovala příslušnou závislost (tj. závislost výše poplatku na číselné hodnotě hmotnosti psaní) a sestrojte její graf.

Fig. 7: Task in Figure 3

Příklad 9

Určete, po kolika letech bude na účtu částka vyšší než 1 000 000 Kč, jestliže si uložíme částku 500 000 Kč na 4% úrok p.a. při složeném úrokování (pro jednoduchost neuvažujte daň z úroků).

řešení

1. krok
Při složeném úrokování připisuje banka na konci každého úrokovacího období vkladateli úrok a v následujícím období se úrok vypočítává z takto navýšené částky.

2. krok
V čase $t = 0$ (na začátku) máme uloženo 500 000 Kč.

3. krok
V čase $t = 1$ (na začátku druhého roku) máme uloženo $500\,000 \cdot 1{,}04 = 520\,000$ Kč.

4. krok
V čase $t = 2$ (na začátku třetího roku) máme uloženo $500\,000 \cdot 1{,}04^2 = 540\,800$ Kč.

5. krok
Označme x hledaný počet úrokovacích období (let). Dostáváme tak nerovnici:

$$500\,000 \cdot 1{,}04^x \geq 1\,000\,000$$

6. krok
Upravíme na tvar:

$$1{,}04^x \geq 2$$

7. krok
Zlogaritmujeme obě strany nerovnice logaritmem o základu 10. Vzhledem k tomu, že logaritmus o základu 10 je rostoucí funkce, zůstane nerovnost zachována a dostaneme:

$$x \cdot \log 1{,}04 \geq \log 2$$

$$x \geq \frac{\log 2}{\log 1{,}04} \doteq 17{,}67$$

závěr
Po 18 letech bude na účtu více než 1 000 000 Kč.

Fig. 8: Task in Figure 4