Quadrilateral definitions in the Merriam-Webster dictionary: Examining the relationships among quadrilaterals

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Definitions are of critical importance in both mathematics and mathematics learning. Online dictionaries have become a commonly used resource for students and teachers seeking mathematical definitions. In this context, it is important to examine how mathematical concepts are defined and presented in such publicly accessible platforms. Given its longstanding presence and widespread use, the U.S.-based Merriam-Webster Dictionary was selected for the current study. We examined the definitions of quadrilaterals provided in both the default (Primary) section and the Student Dictionary for Kids. The results indicated that the definitions in the "Kids Definition" section were generally longer and often included superfluous information compared to those in the "Primary" section. Among the special quadrilaterals presented, most were defined in relation to one another, supporting a structure that allows for the identification of subsets within broader quadrilateral types.

Key words: definitions, quadrilaterals, online dictionary, Merriam-Webster Dictionary.

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1 Introduction

According to Alcock and Simpson (2017, p. 5), "definitions are central to contemporary formal mathematics because, in order to develop deductive arguments and to communicate clearly, mathematicians need to agree upon precise meanings for mathematical concepts". For instance, definitions specify the distinguishing characteristics of a concept, constitute the foundational components of concept formation, serve as a basis for proof and problem-solving, and facilitate the communication of mathematical ideas by promoting consistency in the meaning of concepts (Zaslavsky & Shir, 2005). For any given mathematical concept, there may be equivalent definitions. The choice among these equivalent definitions may vary depending on personal preference, convenience, or contextual considerations. However, the availability of such options presupposes an awareness of their logical equivalence (Van Dormolen & Zaslavsky, 2003). From a mathematics education perspective, teachers may select definitions based on both pedagogical (e.g., intuitive appeal, alignment with students' needs, clarity, and accessibility for learners) and mathematical (e.g., accuracy) characteristics (Leikin & Winicky-Landman, 2000, 2001).

In the past, individuals may have relied on textbooks, printed dictionaries and encyclopedias to look up mathematical definitions. With the advancement of technology, it has become more common to use online resources, such as artificial intelligence tools and online dictionaries, for the same purpose (Kissane, 2008; Patterson & Young, 2013). Some historical dictionaries now offer online versions to meet new demands, such as increased accessibility. Stakeholders in education (e.g., teachers, students, and parents) may use online dictionaries, for instance, to look up mathematical definitions (Harper et al., 2021; Kissane, 2008; Patterson & Young, 2013). Also, "mathematics textbooks at the elementary school level often follow the standard dictionary conception of definition" (Usiskin & Griffin, 2008, p. 3). So, the influence of dictionaries in mathematics education is widespread. However, writing definitions in mathematics requires expertise in the field. Texts and scripts written by mathematics educators and teachers are not always error-free (Usiskin & Griffin, 2008).

In the present study, we focused on the Merriam-Webster Dictionary – "an Encyclopaedia Britannica company, [that] has been America's leading provider of language information for more than 180 years" (Merriam-Webster, n.d.-a, para. 1). The dictionary includes a general definition section – referred to in this study as the "Primary" definition (Merriam-Webster, n.d.-b) – which appears by default when a word is entered, as well as a "Kids Definition" section (Merriam-Webster, n.d.-c) specifically designed for younger users. We analyzed the definitions of quadrilaterals in both sections, given the longstanding interest in mathematics education research in the precision and clarity of such definitions, as well as in the hierarchical relationships among quadrilaterals (e.g., viewing a square as a specific type of rectangle and defining it as such) (de Villiers, 1994; Jones, 2000; Usiskin & Griffin, 2008).

We concur that, in formal mathematics, it is a general standard that a definition does not contain superfluous information namely, it should be minimal (Usiskin & Griffin, 2008; Zaslavsky & Shir, 2005). Therefore, formal mathematics dictionaries such as the Concise Oxford Dictionary of Mathematics (Clapham & Nicholson, 2009) should adhere to this principle. On the other hand, it is difficult to expect formal mathematical definitions in the Merriam-Webster Dictionary, given its wide intended audience

(Merriam-Webster, 2026). Therefore, one may reasonably encounter colloquial definitions of mathematical terms or characterization-like definitions with superfluous information in dictionaries intended for a broader range of users. Embracing this belief, in the present study, we aimed to address the following research question: In what ways are quadrilaterals defined in the "Primary" and "Kids Definition" sections of the Merriam-Webster Dictionary?

1.1 Rationale for research

According to Zazkis and Leikin (2008), "the definition of a concept, once determined in a curriculum, influences the approach to teaching mathematics, the learning sequence, and the set of theorems and proofs" (pp. 132–133). Much mathematics education research examines definitions of mathematical concepts, particularly quadrilaterals, in textbooks (Abdullah & Shin, 2019; Avcu, 2019; Usiskin & Griffin, 2008). Beyond textbooks, online resources, especially dictionaries, have become increasingly popular for accessing mathematical definitions due to their frequent updates and vast information availability (Kissane, 2008; Patterson & Young, 2013). Given their expanding role in learning, investigating the mathematical and pedagogical characteristics of online dictionaries, addressing limitations, and updating them is vital. However, little research addresses how mathematical concepts are defined in dictionaries.

In the present study, Merriam-Webster is selected as the online dictionary for examining definitions due to its widespread use, high visibility in search engine results, and dual-format offerings. Notably, Merriam-Webster provides both a Primary (general) definition section (Merriam-Webster, n.d.-b) and a "Kids Definition" section tailored for student use (Merriam-Webster, n.d.-c). These sections allow for an analysis of how mathematical ideas are communicated differently to adult and young audiences, which is of pedagogical importance. Since students and mathematics teachers may rely on online searches rather than textbooks for immediate access of definitions, understanding how widely accessible dictionaries present mathematical content is critical. Moreover, as definitions presented in such public resources may shape learners' conceptual understandings, it becomes essential to investigate whether those definitions reflect mathematical accuracy, coherence, and instructional usability. By examining both the mathematical precision and pedagogical clarity of Merriam-Webster's definitions, this study aims to highlight affordances and limitations of such resources in supporting students' understanding of geometric concepts.

Definitions and classifications are interrelated notions in geometric investigations, and together they constitute one of the *big ideas* in school geometry (Sinclair et al., 2012). Classifying and defining geometric objects also provide rich contexts for the development of mathematical reasoning (National Council of Teachers of Mathematics [NCTM], 2000). In this regard, the topic of quadrilaterals provides rich content for both geometric research and the development of mathematical reasoning due to their inclusion of various equivalent statements and alternative classification approaches (de Villiers, 1994; de Villiers et al., 2009; Van Dormolen & Zaslavsky, 2003). Moreover "there is some disagreement in the definitions and, consequently, in the ways in which quadrilaterals are classified and relate to each other" (Usiskin & Griffin, 2008, p. X). Presenting definitions of quadrilaterals to students in classrooms is not a straightforward task for teachers (Avcu, 2023). In this respect, our study, which approaches the definitions and classifications of quadrilaterals in online dictionaries from various perspectives, may contribute to ongoing discussions.

1.2 Theoretical background

As de Villiers et al. (2009, p. 193) emphasized, "a definition that contains conditions (properties) that are both necessary and sufficient is said to be correct". To characterize a correct definition, "it is helpful to recall that logically in the biconditional $p \Leftrightarrow q$, the condition p is viewed as necessary and sufficient for the condition q, meaning that one can conclude that q follows from p, and vice versa" (pp. 193–194). For instance, one may mistakenly define a rhombus as "a convex kite with congruent diagonals that bisect each other" in which a rhombus is depicted as a subset of a kite. However, this definition inadvertently characterizes a special rhombus, namely a square, because having congruent diagonals is not a necessary property of all rhombi. In another example, the definition "a square is a parallelogram with congruent diagonals" includes a necessary property of squares. However, the definition is not sufficient to define a square, as "congruent diagonals" also applies to rectangles with non-congruent adjacent sides. Thus, having congruent diagonals is a necessary but not sufficient condition for defining a square. As is evident, "a definition is incorrect if it contains insufficient or unnecessary properties" (de Villiers, 2009, p. 194).

Simply defined, a quadrilateral is a four-sided polygon (Usiskin & Griffin, 2008). Here, the use of the word 'polygon' connects several essential properties within a single term (Pereira-Mendoza, 1993). Zazkis and Leikin (2008) highlight the use of inaccurate terminology such as a figure and shape when defining special quadrilaterals. For example, according to Zazkis and Leikin (2008, p. 139), defining a square as "a four-sided figure with four equal sides and four equal angles" is an appropriate but non-rigorous example of a definition. The researchers interpreted that "a 'four-sided figure' or 'shape' implies a polygon", even though such a definition of a square does not make explicit that the sides are composed of coplanar and congruent line segments joined to one another at right angles. Arguably, when sides are referenced in definitions of quadrilaterals, the term evokes polygons. In this context, sides in plane geometry refer to coplanar, conjoined line segments that form a closed figure, thereby implying a polygon – even when non-rigorous terminology is used in definitions of quadrilaterals. Therefore, the use of terms such as shapes or figures in defining quadrilaterals may lead to definitions that are appropriate in context with some non-rigorous examples (Zazkis & Leikin, 2008).

A definition of a special quadrilateral may emphasize different aspects of its properties and establish various relationships among quadrilaterals. For instance, a definition may use minimal properties without including superfluous information. It may also define a special quadrilateral in relation to another, treating it as a subset. In the following subsections, we elaborate on the various aspects of definitions used to characterize relationships among special quadrilaterals.

1.2.1 Partitional and hierarchical definitions among quadrilaterals

The ways in which quadrilaterals have been associated with one another and defined have evolved over the centuries. For instance, in the early development of geometry, Euclid defined a rhombus as "of quadrilateral figures, . . . a rhombus (from Greek: $\rho\mu\beta o\sigma$) that which is equilateral but not right-angled" (Usiskin & Griffin, 2008, p. 19). This definition disjoins squares from rhombi, exemplifying a partitional definition, in which "the concepts involved are considered disjoint from each other" (de Villiers et al., 2009, p. 191). On the other hand, defining a rhombus as "a quadrilateral with congruent sides" establishes a *hierarchical* (inclusive) relationship between a square and a rhombus. Here, the square is accepted as a special case of the rhombus, unlike Euclides' *partitional* (exclusive[1]) definition. As de Villiers (1994, p. 11) emphasized, "by the term hierarchical classification is meant here the classification of a set of concepts in such a manner that the more particular concepts form subsets of the more general concepts". In a similar vein, Usiskin and Griffin (2008, p. 6) pointed out that "an inclusive definition creates a link in a hierarchical chain from the more general to the more specific".

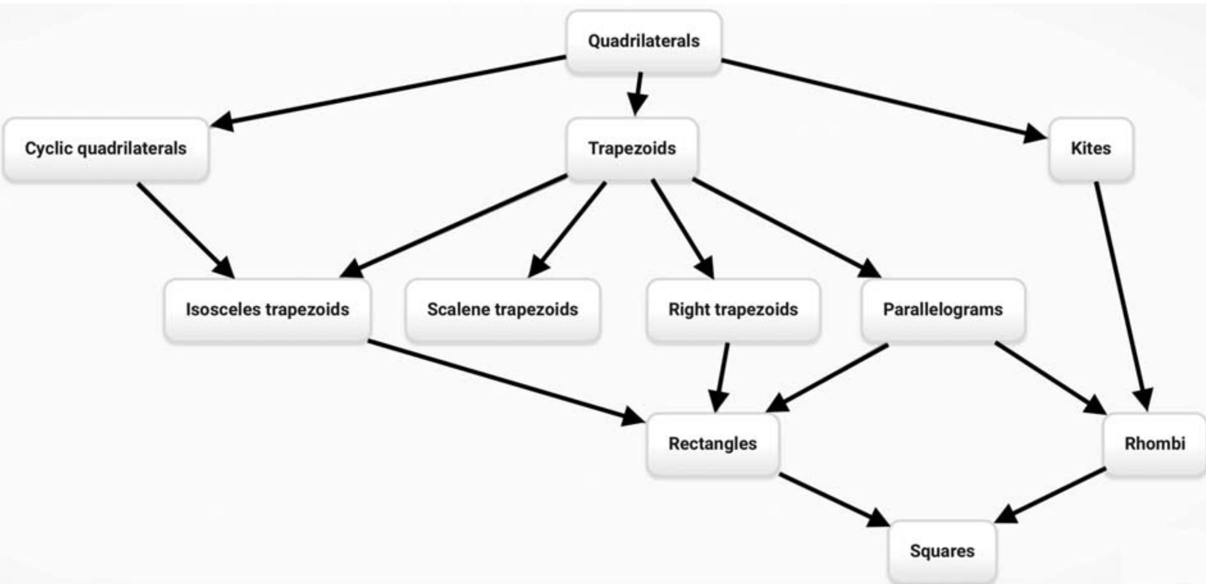

Fig. 1: A hierarchical classification of some quadrilaterals[2]

Partitional definitions can also be made for more general concepts such as quadrilaterals and polygons. For instance, some resources may define a quadrilateral as "the union of four line segments that join four coplanar points, no three of which are collinear, each segment intersecting exactly two others, one at each endpoint" (Usiskin & Griffin, 2008, p. 11). This definition excludes crossed quadrilaterals[3] (de Villiers, 1994). Drawing on Usiskin and Griffin's (2008) study of definition in geometry, Fig. 1 presents a hierarchi-

[1] "An exclusive definition creates a partition of the more general object into a set of more specific objects" (Usiskin & Griffin, 2008, p. 6).

[2] Our classification focuses on convex quadrilaterals, defined as quadrilaterals whose diagonals intersect at a point inside the figure (Graumann, 2005).

[3] "A crossed quadrilateral is a quadrilateral with two of the sides also crossing each other at a point other than the vertices" (de Villiers, 1994, p. 13).

cal classification of the most common special quadrilaterals[4] taught in K–16 mathematics. In Fig. 1, an arrow indicates that the special quadrilateral below is *always* considered a special case of the quadrilateral above. Some mathematics educators also characterize interrelationships between two quadrilaterals using the adverbs *always* and *sometimes* to indicate the hierarchical direction (Duatepe-Paksu & Žilková, 2018; Knapp et al., 2007). For instance, by definition, cyclic quadrilaterals are quadrilaterals whose vertices lie on a circle (Usiskin & Griffin, 2008). Since all isosceles trapezoids, rectangles, and squares can be inscribed in a circle, they are special cases of cyclic quadrilaterals. Let us characterize the hierarchical direction between a cyclic quadrilateral and a rectangle. According to Fig. 1, a cyclic quadrilateral is *sometimes* a rectangle, because cyclic quadrilaterals form a broader class, and only those satisfying the additional condition of having four right angles are rectangles. On the other hand, a rectangle is *always* a cyclic quadrilateral, since the vertices of a rectangle always lie on a circle. The hierarchical classification makes it possible to define a quadrilateral using others that appear higher in the hierarchy (Zazkis & Leikin, 2008).

Some quadrilateral definitions remain debatable, while others are more widely agreed upon. As Usiskin and Griffin (2008, p. 26) emphasized, "there is no disagreement among today's textbook authors regarding which special types of quadrilaterals are always parallelograms. Rectangles, rhombuses, and squares are universally viewed as parallelograms". On the other hand, for a long time, textbooks defined a trapezoid (known as a *trapezium* in British English) as "a quadrilateral with exactly one pair of parallel sides" (Usiskin & Griffin, 2008, p. 27). This definition does not associate a trapezoid with a square, rectangle, rhombus, or parallelogram; therefore, it is a partitional definition.

Some researchers or curriculum writers may still prefer a partitional definition of a trapezoid over a hierarchical one (Casa & Gavin, 2009). However, it is now increasingly common to define a trapezoid hierarchically as "a quadrilateral with at least one pair of opposite parallel sides" (Usiskin & Griffin, 2008, p. 27), making it possible to classify parallelograms, rectangles, rhombi, and squares as special types of trapezoids (Jones, 2000). Using a partitional definition of a trapezoid may not be considered incorrect, but rather idiosyncratic, and "thus the decision one makes in choosing a definition for trapezoid is precisely whether one wishes to include parallelograms in the trapezoid family" (Usiskin & Griffin, 2008, p. 29). Yet, according to de Villiers (1994), hierarchical definitions should be preferred over partitional ones, as they offer both pedagogical and logical advantages.

A hierarchical definition of a special quadrilateral can be expressed concisely, foregrounding only the necessary and sufficient properties. Alternatively, it may include several properties, resulting in a relatively wordy statement. Both types of definitions are common and offer different mathematical and pedagogical affordances. The next section addresses these differences within hierarchical definitions.

1.2.2 Economical and uneconomical hierarchical definitions

In hierarchical definitions that relate quadrilaterals to one another, superfluous properties may be included in addition to those that are necessary and sufficient. As de Villiers et al. (2009) explained, "an economical definition has a minimal set of necessary and sufficient properties; that is, it has no superfluous information. Conversely, an uneconomical definition contains redundant properties" (p. 196). For instance, an uneconomical definition of a rectangle is: "a rectangle is a parallelogram with congruent diagonals and four right angles." The definition is hierarchical, as it also applies to squares, which are considered subsets of rectangles. However, once the right angles are specified, mentioning the congruent diagonals becomes redundant. Therefore, the definition is considered uneconomical.

Zazkis and Leikin (2008) pointed out that the criteria for minimality in definitions remain debatable (see Avcu, 2019; de Villiers et al., 2009; Van Dormolen & Zaslavsky, 2003; Zaslavsky & Shir, 2005). For instance, according to de Villiers et al. (2009, p. 198), defining a rhombus "as a quadrilateral with four congruent sides" is economical "insofar as the defining conditions contain no superfluous information". Yet, for some, referring to the number of sides in the definition is redundant, given that the definition specifies a quadrilateral. In this context, a minimal definition would be: "a rhombus is a quadrilateral with congruent sides." As Usiskin and Griffin (2008, p. 3) underscored, "sometimes authors insert redundant distinguishing characteristics to make it easier for students to deduce properties of the object". On the other hand, according to Zaslavsky and Shir (2005, p. 320) –

> a minimal definition should consist only of information that is strictly necessary for identifying the defined concept. For example, defining a rectangle as a *quadrangle with four right angles* is not a minimal definition, since it is enough to require that there be *three right angles.*

However, this definition requires a deduction to verify that the fourth angle is also a right angle (Jamison, 2000). Thus, while de Villiers et al. (2009) emphasize pedagogical aspects in their approach to

[4]For more types of specials quadrilaterals, see Graumann (2005).

economical definitions, recognizing that different definitions may be preferable for instructional purposes, Zaslavsky and Shir (2005) focus on logical (deductive) aspects, aiming for definitions that are minimal in the strict logical sense. Elsewhere, Van Dormolen and Zaslavsky (2003, p. 96) further elaborated on this distinction as follows:

> On first sight the criterion of minimality seems to be more of an aesthetic or philosophical nature than a logical one. Indeed, describing a rectangle as quadrilateral with four right angles will not result in a contradiction in the system, and may have advantages from a pedagogical perspective. Moreover, often at the time when a concept is defined there may not be sufficient knowledge to determine whether it is minimal. Thus, insisting on this criterion may impede the development of certain concepts or theories.

An economical or uneconomical hierarchical definition may further emphasize different relationships among quadrilaterals both in terms of identifying subsets of the defined concept and in the use of deductions (e.g., logical arguments). The next section elaborates on a fine-grained classification of economical and uneconomical hierarchical definitions.

1.2.3 Subcategorical and deductive relationships within economical and uneconomical hierarchical definitions

An economical or uneconomical definition of a special quadrilateral can be made directly from one of the broadest relevant categories, namely polygons, quadrilaterals or figures/shapes. For instance, a rhombus can be defined as "a quadrilateral with congruent sides and perpendicular diagonals".[5] In this uneconomical definition, the rhombus is defined by means of a general concept of its type, quadrilateral. In another example, the economical definition "a rhombus is a four-sided polygon with congruent sides", uses another general concept 'polygon' and specifies it with its number of sides and properties. Note both definitions are hierarchical, encompassing squares as special cases of rhombi.

Alternatively, what we call a *subcategorical* definition, a special quadrilateral is defined in terms of another special quadrilateral. For instance, a rhombus can be defined by means of kites, parallelograms, or trapezoids (Fig. 1), as in the economical definition: "a rhombus is a trapezoid with congruent sides," or in the uneconomical definition: "a hombus is a parallelogram with perpendicular diagonals that bisect one another." The use of a family relationship within a hierarchy of properties can facilitate "the deductive systemization and derivation of properties of more special concepts" (de Villiers, 1994, p. 15). For example, the subcategorical and economical definition "a square is a rhombus with a right angle" requires establishing that one right angle in a rhombus necessitates that the other three angles are also right angles (Jamison, 2000). When students encounter this definition, they may question the measures of the rest of the angles in the rhombus, since the definition refers to *a right angle*. According to Jamison (2000), such definitions should be dispreferred. Regarding the subcategorical-economical definition "a rectangle is a *parallelogram* whose diagonals have equal lengths" (Jamison, 2000, p. 49), Jamison claimed that the statement is in the form of a theorem to be proved. He posited:

> This statement is true and concise, but the defining property is not BASIC. This would work better as a theorem to be proved than as a definition. In mathematics, assertions of this kind are regarded as *characterizations* rather than as definitions.

Jamison (2000, p. 48) contended that "a definition is a *concise* statement of the *basic* properties of an object or concept which *unambiguously identify* that object or concept... It should involve *basic* properties, ideally those that are simply stated and have immediate intuitive appeal". In this context, Jamison does not appreciate theorem-like definitions "that require extensive derivation or are hard to work with" such as using diagonal properties (e.g., diagonals that bisect each other, and perpendicular diagonals) to deduce congruent sides or angles in a quadrilateral. Then, making a subcategorical definition may be challenging for students, since it necessitates associating a special quadrilateral with another one, which increases complexity of definitions compared to defining a special quadrilateral by means of a polygon or a quadrilateral (Jamison, 2000).

On the other hand, de Villiers et al. (2009, p. 199) referred to convenient definitions, stating: "a good, or convenient, definition is one that also allows us to deduce the other properties of the concept easily; that is, it should be *deductive-economical*". Deductive properties necessitate the use of theorems or other properties (e.g., congruence), requiring proofs or arguments in quadrilateral definitions (de Villiers et al., 2009; Van Dormolen & Zaslavsky, 2003). For example, de Villiers et al. (2009, p. 199) provided the following definition of a rhombus as "a quadrilateral with two axes of symmetry through the opposite

[5] Definitions in quotation marks without citations are those provided by the authors of this study.

angles". This economical definition of a rhombus draws on symmetry, which must relate to congruency in order to deduce its properties. Consistent with this view, Usiskin and Griffin (2008, p. 57) stated the theorem "every rhombus is symmetric to the lines containing its diagonals", thereby treating symmetry as a consequence to be proved, rather than as an assumed defining property.

In a similar vein, other types of definitions (e.g., subcategorical, uneconomical) may also possess deductive qualities. For instance, the definition "a rectangle is an isosceles trapezoid with congruent diagonals that bisect each other" includes superfluous information – specifically, the term "congruent" is redundant, as an isosceles trapezoid always has congruent diagonals. Therefore, this is considered an uneconomical definition. Additionally, it is subcategorical, as it defines a rectangle by associating it with an isosceles trapezoid. This definition also requires the use of the diagonal properties of the isosceles trapezoid in the sense of diagonal bisection – a property not shared by all isosceles trapezoids. Therefore, it involves deduction and is thus characterized as *deductive–subcategorical–uneconomical*.

Overall, Fig. 2 presents a chart that depicts the types of hierarchical definitions, accompanied by sample definitions within the classification of quadrilaterals. A hierarchical definition for quadrilaterals can be formulated either economically or uneconomically. An economical definition from a *pedagogical* perspective (de Villiers et al., 2009) uses the necessary and sufficient properties, though it is not necessarily strictly minimal from a logical perspective, as emphasized by Van Dormolen and Zaslavsky (2003). In contrast, an uneconomical definition includes superfluous information, even though it still contains the necessary and sufficient properties of the defined concept (de Villiers et al., 2009). If an economical or uneconomical definition uses one special quadrilateral to define another, it is further characterized as a subcategorical definition. If the definition involves deducing properties (e.g., through proofs or logical arguments), it is further categorized as a deductive definition. Finally, if an economical or uneconomical definition incorporates both aspects, it is considered both subcategorical and deductive.

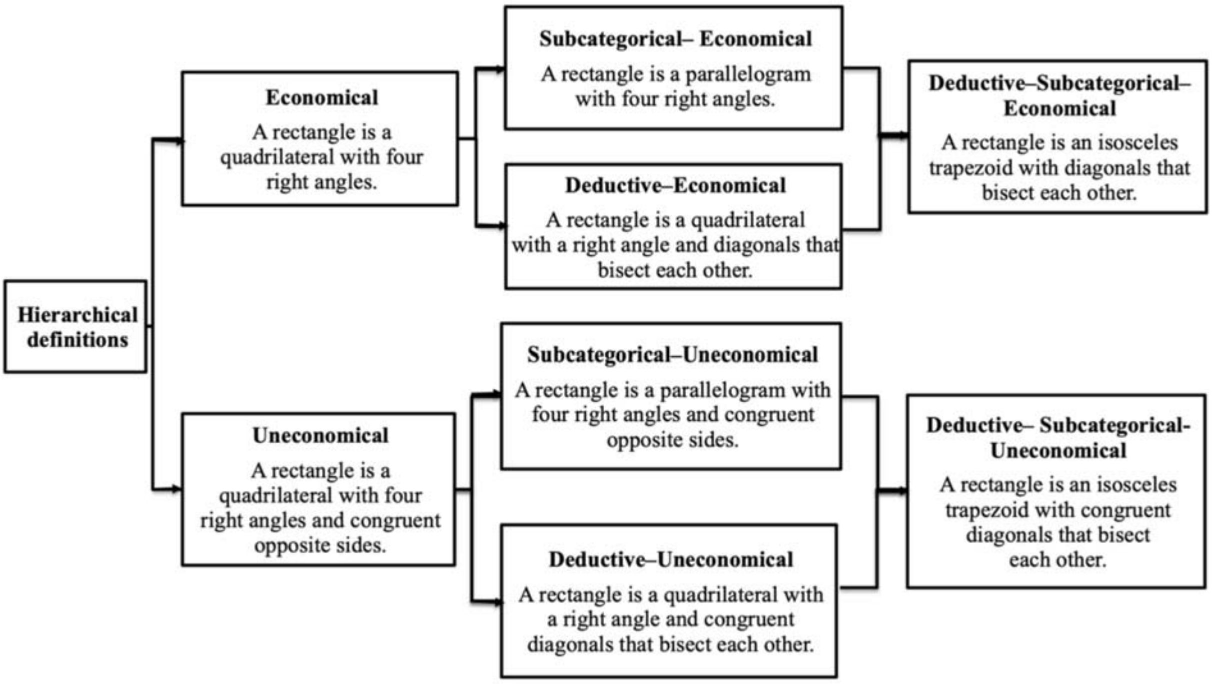

Fig. 2: Types of hierarchical definitions and sample definitions

2 Methods

We first searched for popular online dictionaries and excluded commercial dictionaries (e.g., Oxford English Dictionary) that do not provide free access to users. We selected the U.S.-based Merriam-Webster Dictionary due to its online accessibility and long-standing historical presence (Merriam-Webster, n.d.-a). From a hierarchical classification perspective (Fig. 1), we examined the most common special quadrilaterals taught in K–16 mathematics. In this context, we searched for the following geometric shapes: cyclic quadrilateral, isosceles trapezoid, kite, parallelogram, quadrilateral, rectangle, rhombus, right trapezoid, scalene trapezoid, square, trapezium, and trapezoid. However, the *mathematical* definitions found in the dictionary was limited to rhombus, parallelogram, quadrilateral, rectangle, rhombus, square, trapezium, and trapezoid.

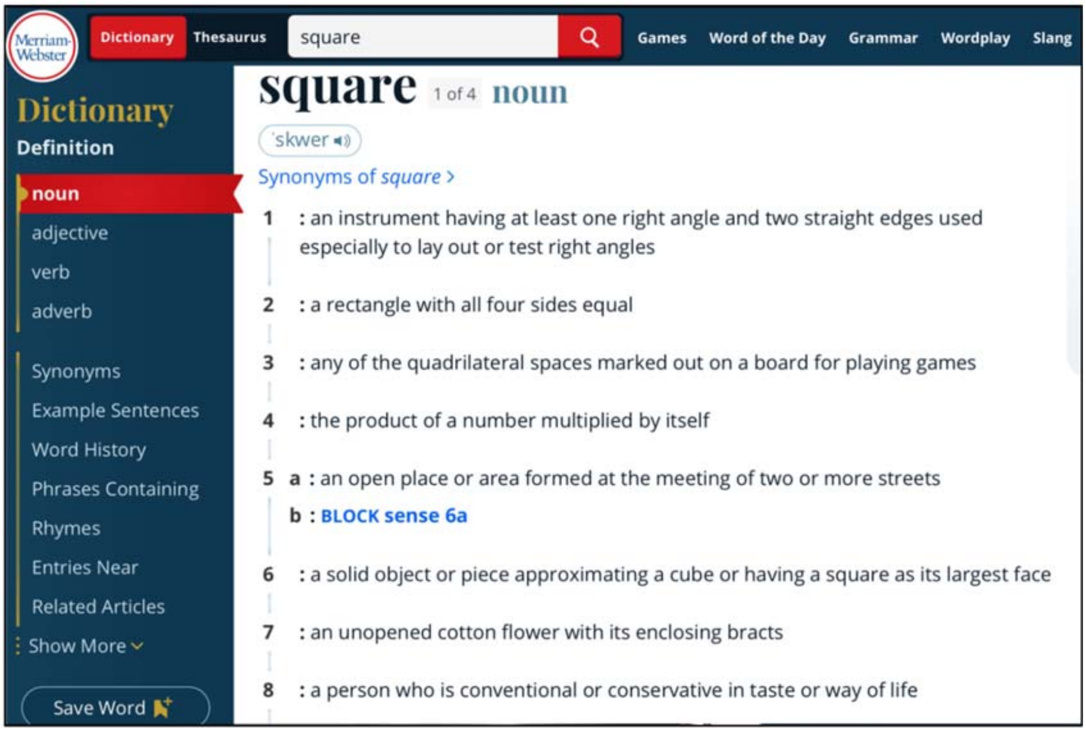

Fig. 3: Multiple noun definitions of square in the Primary Definition section

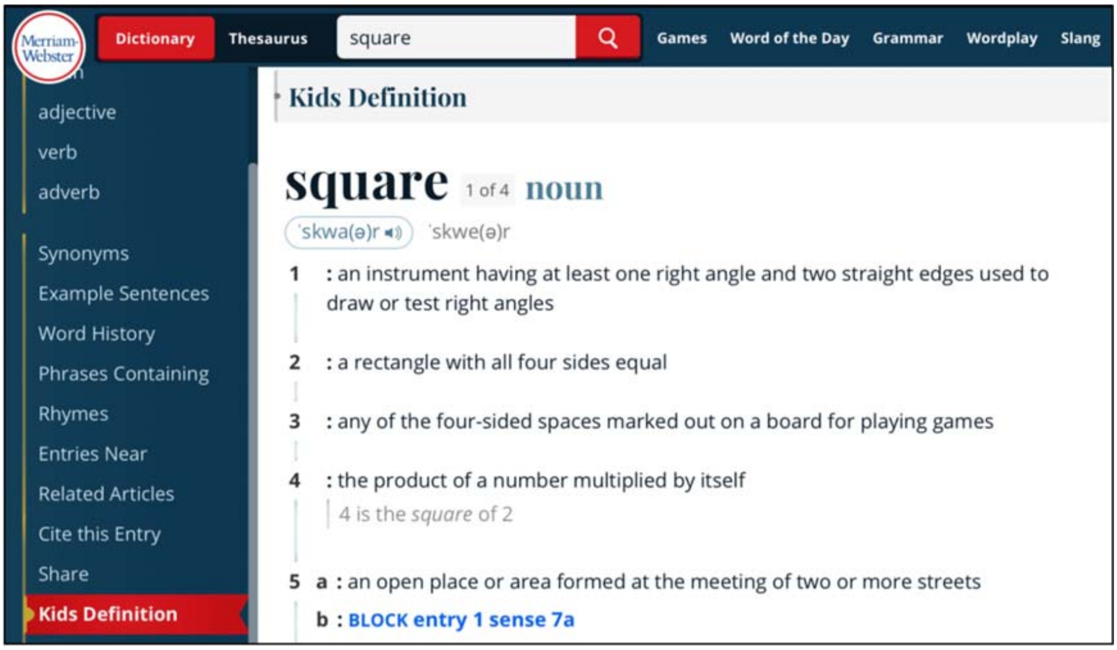

Fig. 4: Multiple noun definitions of square in the "Kids Definition" section

When a word is looked up in the Merriam-Webster Dictionary, the "Primary" Definition section (Merriam-Webster, n.d.-b) provides its noun, adjective, verb, and adverb meanings, if applicable (Fig. 3). For our study, we examined the mathematical definitions listed under the noun form. Among the noun definitions, we identified mathematics-related definitions of the concept when multiple meanings existed (e.g., colloquial uses). For instance, Fig. 3 presents several definitions of a square. Only the second definition pertains to the quadrilateral. The sixth definition, although related to mathematics, concerns solid geometry and was therefore excluded from our study.

The dictionary also includes a "Kids Definition" section, described as: "Search an online dictionary written specifically for young students. Kid-friendly meanings from the reference experts at Merriam-Webster help students build and master vocabulary" (Merriam-Webster, n.d.-c, para. 1). We also searched for quadrilaterals (e.g., Fig. 4) in the "Kids Definition" section and applied a similar selection process to identify relevant definitions for our study, as was done in the "Primary" definition section.

2.1 Coding and data analysis

We analyzed the mathematical definitions of rhombus, parallelogram, quadrilateral, rectangle, rhombus, square, trapezium, and trapezoid in both sections of the Merriam-Webster Dictionary. Since the online definitions may change over time, we provided screenshots of the analyzed definitions in our data presentation. Each definition was evaluated to determine whether it accurately provided the necessary and sufficient conditions for the geometric shape. Definitions that failed to meet these conditions were categorized as incorrect. If a definition disjointed a special quadrilateral from others, it was coded as partitional. Finally, if a definition provided the necessary and sufficient properties, leading to hierarchical relationships among quadrilaterals, it was classified as hierarchical.

Hierarchical definitions were further categorized based on whether they contained superfluous information; in particular, we identified whether they were formulated economically or uneconomically. We adopted the principle of minimality in definitions, as advocated by de Villiers et al. (2009), emphasizing pedagogical advantages rather than aesthetic or philosophical considerations, as discussed by Van Dormolen and Zaslavsky (2003). Our decision to adopt a pedagogical perspective was particularly influenced by our analysis of the "Kids Definition" section.

Finally, we examined whether an economical or uneconomical hierarchical definition incorporated deductive properties and whether one type of special quadrilateral was used to define another. For instance, a rhombus can be defined by means of a special quadrilateral (trapezoid, parallelogram, or kite) or a more general concept (e.g., quadrilateral, polygon, or geometric shape). If it is defined by means of a special quadrilateral, we coded it as subcategorical, as in the following economical definition: "a rhombus is a parallelogram with adjacent congruent sides." Therefore, the definition is subcategorical-economical.

In another example, the economical definition "a rhombus is a quadrilateral with at least two lines of symmetry" incorporates a deduction-based argument, as it entails applying a reflection transformation to quadrilaterals and deducing that any quadrilateral with two or four lines of symmetry meets the criteria, given the requirement of "at least two lines of symmetry." Therefore, it is categorized as deductive-economical. If both deductive and subcategory-based elements are present in an economical definition, as in the definition "a rhombus is a kite with at least two lines of symmetry," it is categorized as deductive-subcategorical-economical. A similar coding scheme was applied to uneconomical definitions.

Table 1 presents the codebook used in our data analysis. We categorized each quadrilateral according to the coding scheme, and constructed data matrices for both sections of the dictionary. In total, we

Table 1: Codebook of data analysis

Definition type	Description	Sample definition
Incorrect	The definition does not have the necessary and/or sufficient properties of the concept.	A rhombus is a parallelogram with all right angles.
Partitional	The definition disjoints a special quadrilateral from other types, not leading to hierarchical relationships among quadrilaterals.	A rhombus is a quadrilateral with equal side lengths and no right angles.
Hierarchical		
Uneconomical	The definition with the necessary and sufficient properties of the concept provides superfluous information.	A rhombus is a quadrilateral with congruent sides and perpendicular diagonals.
Economical	The definition with the necessary and sufficient properties of the concept provides no superfluous information.	A rhombus is a quadrilateral with congruent sides.
Deductive-Uneconomical	An uneconomical definition that also allows for deducing properties of the concept by using theorems or other properties, requiring proofs or logical arguments in quadrilateral definitions.	A rhombus is a quadrilateral with perpendicular diagonals and at least two lines of symmetry.
Deductive-Economical	An economical definition that also allows for deducing properties of the concept by using theorems or other properties, requiring proofs or logical arguments in quadrilateral definitions.	A rhombus is a quadrilateral with at least two lines of symmetry.
Subcategorical-Uneconomical	The definition establishes a classification in which a more specific concept is associated with a more general one with superfluous information.	A rhombus is a parallelogram with adjacent congruent sides and perpendicular diagonals.
Subcategorical-Economical	The definition establishes a classification in which a more specific concept is associated with a more general one with no superfluous information.	A rhombus is a parallelogram with adjacent congruent sides.
Deductive-Subcategorical-Uneconomical	A subcategorical-uneconomical definition that also allows for deducing properties of the concept by using theorems or other properties, requiring proofs or logical arguments in quadrilateral definitions.	A rhombus is a kite with adjacent congruent sides and at least two lines of symmetry.
Deductive-Subcategorical-Economical	A subcategorical-economical definition that also allows for deducing properties of the concept by using theorems or other properties, requiring proofs or logical arguments in quadrilateral definitions.	A rhombus is a kite with at least two lines of symmetry.

coded seven definitions in the "Primary" section and six definitions in the "Kids Definition" section. First, the authors coded the definitions in the "Kids Definition" section individually. Then, the authors met and compared their coding, noting any differences. In this process, the coding scheme was revisited to ensure consistent interpretation and application of the categories through ongoing discussion. For instance, a rhombus is defined as "a parallelogram with all four sides of equal length and usually with no right angles" in the "Kids Definition" section. One of the authors thought this definition suggested a relatively partitional definition since the adverb 'usually' was used, which might imply not associating a rhombus with a square. The other author pointed out that the definition did not prevent classifying a square as a subset of a rhombus. The authors reached a consensus that the use of the adverb 'usually' did not exclude a square from being classified as a rhombus. Therefore, the definition was not categorized as partitional. A similar coding negotiation was used in the "Primary" section of the dictionary. Finally, we compared the definitions across the "Primary" and "Kids Definition" sections.

3 Results

Across the dictionary sections, definitions of cyclic quadrilaterals, kites, isosceles trapezoids, and scalene trapezoids are not presented in either section. Table 2 summarizes the types of definitions found in the "Primary" and "Kids Definition" sections of the dictionary. A partitional definition of 'trapezoid' is presented in both sections. In the "Primary" section, 'trapezoid' is defined as "a quadrilateral having only two sides parallel," while the Kids Definition section states, "a polygon that has four sides and exactly two that are parallel" (Fig. 5). Notably, the latter definition is longer and defines a trapezoid in terms of a polygon. In both cases, 'trapezoid' is defined using partitional classifications, excluding the parallelogram family from being considered subsets of trapezoids. Moreover, the definition of 'trapezium' in the "Primary" section as "a quadrilateral with no sides parallel" is incorrect.

Table 2: Definition types across the dictionary sections

Types of definitions	Primary	Kids Definition
Incorrect	Trapezium	–
Partitional	Trapezoid	Trapezoid
Hierarchical		
Uneconomical	Parallelogram	Parallelogram, Rectangle, Quadrilateral
Economical	Quadrilateral	–
Subcategorical-Uneconomical	Rhombus	Rhombus
Subcategorical-Economical	Square, Rectangle	Square

Fig. 5: The definitions of trapezoid/trapezium across the sections of the dictionary

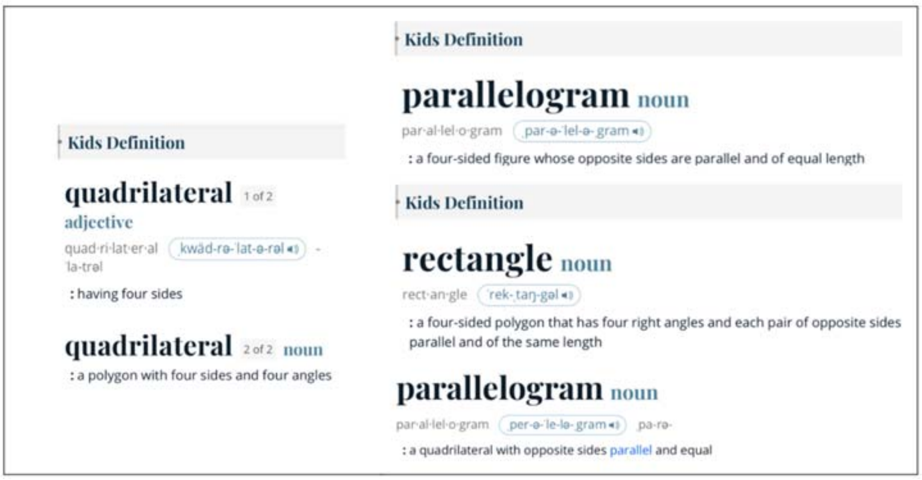

Fig. 6: The uneconomical definitions

The definition of 'parallelogram' in the "Primary" section, and those of parallelogram, rectangle, and quadrilateral in the Kids Definition section, are presented uneconomically with superfluous information (Fig. 6). The "Primary" definition of 'parallelogram' reads: "a quadrilateral with opposite sides parallel and equal.” Here, once the parallelism of opposite sides is stated, referring to their equality in length becomes redundant. Additionally, there is a minor language issue in this definition, as the term congruent would be more appropriate than equal when referring to geometric figures.

In the Kids Definition section, the uneconomical definitions of ‘quadrilateral’ and ‘rectangle’ use the term polygon in their descriptions (Fig. 6). For example, in the definition of ‘quadrilateral,’ after stating the number of sides, mentioning the number of angles is superfluous. The uneconomical definition of ‘parallelogram’ in the Kids Definition section reads: “a four-sided figure whose opposite sides are parallel and of equal length.” Although appropriate in context, this definition is non-rigorous, as the term figure implies a polygon, but polygonal properties (e.g., closed and coplanar sides) are not explicitly stated.

Different from the uneconomical definition provided for ‘quadrilateral’ in the Kids Definition section, ‘quadrilateral’ is defined economically, with no superfluous information, in the “Primary” section (Fig. 7). The definition is based on a polygon with four sides.

Fig. 7: Quadrilateral definition in the Primary section

Fig. 8: Subcategorical-uneconomical definitions

‘Rhombus’ in both sections is defined using subcategorical-uneconomical definitions (Fig. 8). The definition of ‘rhombus’ is given by means of a parallelogram, and it reads: “a parallelogram with four equal sides and sometimes one with no right angles.” Here, after referring to equal side lengths, the mention of a parallelogram with non-right angles is redundant. From a hierarchical point of view, by adding “sometimes one with no right angles” to the definition, the statement implies a parallelogram with congruent sides is *sometimes* a non-right-angled rhombus, despite the fact that a parallelogram with congruent sides may have all right angles. In addition, the definition presents a minor language issue, as it uses “equal sides” instead of referring more precisely to “equal side lengths” or “congruent sides.” In the Kids Definition section, ‘rhombus’ is also defined by means of a parallelogram: “a parallelogram with all four sides of equal length and usually with no right angles.” Here, the definition contains superfluous information in its reference to parallelograms “usually with no right angles.” Notably, the adverb ‘usually’ is used, instead of ‘sometimes.’ This word of choice suggests a preference for associating a parallelogram with a non-right-angled rhombus, while treating the case of a square as less preferred or implicitly dispreferred, despite the fact that a square is also a rhombus within the same hierarchical structure.

Finally, the definitions of ‘rectangle’ and ‘square’ in the “Primary” Dictionary, and ‘square’ in the “Kids Definition” section, are subcategorical-economical (Fig. 9). The definition of ‘rectangle’ is given by means of a parallelogram and does not contain superfluous information from a pedagogical viewpoint. It is important to note that the definition includes the clarification “*especially*: one with adjacent sides of unequal length,” which is not part of the core definition. This clarification does not establish a partitional classification between squares and rectangles. Yet it may subtly reinforce the perception of squares as conceptually different from rectangles, despite their inclusion within the definition as a type of rectangle. The definition of ‘square’ in both sections is stated as “a rectangle with all four sides equal.” This definition uses a special quadrilateral (rectangle) with minimal properties; therefore, it is subcategorical-

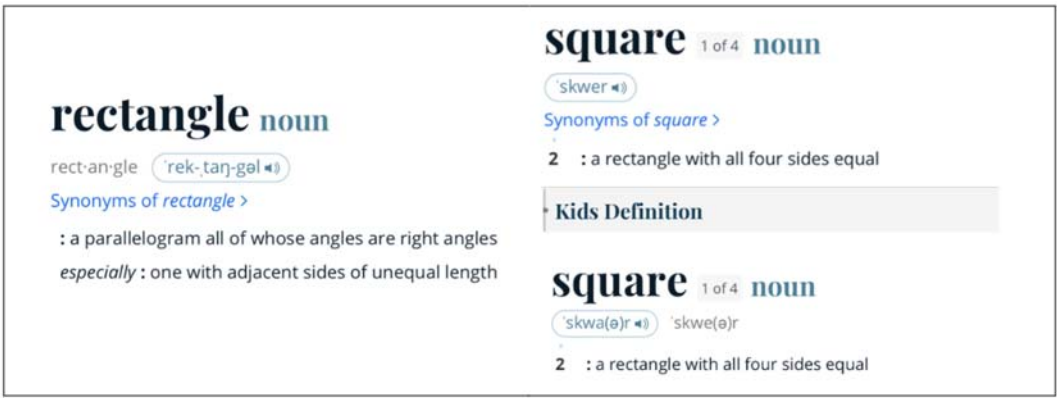

Fig. 9: Subcategorical-economical definitions

economical. A similar language issue appears in the square definition regarding the use of the term equal. More precise wordings could include: four congruent sides, four equal side lengths, or four sides of equal length.

4 Discussion

Online dictionaries have increasingly influenced mathematics education due to their easy access to vast information (Kissane, 2008; Patterson & Young, 2013). While they offer clear benefits for students and teachers, their limitations and areas for improvement warrant attention. Addressing these can raise awareness among practitioners and guide dictionary providers in enhancing their services. Considering the complexities and disagreements surrounding quadrilateral definitions and their instructional challenges (Avcu, 2023; de Villiers, 1994; Usiskin & Griffin, 2008), this study examines in what ways quadrilaterals are defined in the Merriam-Webster Dictionary's "Primary" and "Kids Definition" sections.

We identified seven definitions related to quadrilaterals in the "Primary" section and six in the "Kids Definition" section of the dictionary. Notably, the definition of 'trapezium' is inaccurate, and the definition of 'trapezoid' is framed using a partitional approach. Beyond analyzing the formal aspects of the "Primary" and "Kids Definition" sections in the Merriam-Webster Dictionary (i.e., the types of quadrilaterals included), our study also provides significant insights into the mathematical content of these definitions. A key criterion for evaluating definitions is their accuracy, which involves whether they adequately capture the necessary and sufficient properties of the geometric object (de Villiers et al., 2009).

Hierarchical definitions describe a quadrilateral in terms of another, thereby facilitating an understanding of the relationships among different quadrilaterals (Zazkis & Leikin, 2008). In this regard, they should be preferred over partitional definitions due to their pedagogical and logical advantages (de Villiers, 1994). Our results indicate that both sections of the Merriam-Webster Dictionary predominantly employ hierarchical definitions, which aligns well with the types of definitions taught within the K–16. However, a hierarchical definition for the trapezoid is absent, and thus, the dictionary does not represent the parallelogram family as a subset of trapezoids. Although this omission does not constitute an outright error, revising such definitions to better align with the curricula and textbooks could be considered to enhance consistency and educational relevance.

Two definitions in the "Primary" definitions section (parallelogram and rhombus) and four in the "Kids Definition" section (parallelogram, rectangle, quadrilateral, and square) are identified as uneconomical. The greater level of detail in the "Kids Definition" section, intended to support students' understanding of quadrilaterals, may have contributed to the higher incidence of uneconomical definitions. As addressed earlier, uneconomical definitions in mathematics are often presented in the form of characterizations with superfluous information to help especially young students understand the terms (Jamison, 2000; Usiskin & Griffin, 2008). The publisher (Merriam-Webster, 2026) might have had a similar motive in using characterization-like statements for their readers. Also, in both the "Primary" and "Kids Definition" sections, two approaches to hierarchical definitions are evident. Some specific quadrilaterals are described using broad terms like polygons and shapes (e.g., parallelogram in the Kids section), which risks omitting essential properties and may be pedagogically discouraged (Pereira-Mendoza, 1993). Others are defined by means of special quadrilaterals (i.e., subcategorical definitions), supporting deductive systematization and the derivation of properties related to more specific concepts (de Villiers, 1994). Notably, some definitions have issues with mathematical language, particularly regarding the use of "equal" to describe the relationships between side lengths, where more precise terminology such as "equal side lengths" or "congruent sides" would have been appropriate. It should be noted that providing definitions with imprecise language may impede students' deductive reasoning in the process of creating and critiquing geometric ideas relationships (NCTM, 2000).

Another important result is the absence of deductive definitions in both sections of the dictionary. These definitions, which involve deriving properties through proofs or other geometric attributes (de Villiers et al., 2009; Van Dormolen & Zaslavsky, 2003), can also be embedded within the subcategorial definition types, though they may result in more complex description. Considering that online dictionaries cater to a broad audience with varying mathematical backgrounds, a deductive definition may not have been provided in a lay language, as such definitions presuppose familiarity with additional mathematical properties and relationships that may exceed the background knowledge of many users.

5 Final remarks

Our results revealed that some commonly used types of quadrilaterals in K–16 education (see Fig. 1) were not present in the online dictionary, and even among the defined types, discrepancies were observed between the "Primary" and "Kids Definition" sections. Definitions of cyclic quadrilaterals, kites, isosceles trapezoids, and scalene trapezoids are absent from both sections of the dictionary. While all these quadrilateral types are incorporated within K–16 mathematics, some are introduced in upper grade levels, such as high school (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGA & CCSSO], 2010; NCTM, 2000). The Merriam-Webster Dictionary does not specify the age group targeted by its "Kids Definition" section. Assuming that this section targets young students, the omission of definitions for terms such as cyclic quadrilaterals and kites can be justified on developmental or curricular grounds. However, considering that the "Primary" definitions section is intended for all users and that over 60% of the dictionary's users are aged between 18 and 34 (Merriam-Webster, 2026), it is important that this section comprehensively encompasses all the common hierarchical definitions.

Enriching the dictionary with definitions of various types of quadrilaterals not only meets the needs of diverse users but also reinforces its role as a reliable and accessible resource for both students and professionals. While the Merriam-Webster Dictionary is not a specialized mathematics dictionary, it plays an influential role in shaping public understanding of mathematical terminology, especially due to its wide accessibility and prominence in online search results. According to the publisher –

> Each day most Merriam-Webster editors devote an hour or two to reading a cross section of published material, including books, newspapers, magazines, and electronic publications; in our office this activity is called "reading and marking." ... Any word of interest is marked, along with surrounding context that offers insight into its form and use. (Merriam-Webster, n.d.-d, para. 4)

Given this editorial process, definitions of additional special quadrilaterals may be incorporated into the dictionary in the future. Such updates would inevitably raise important questions about how these geometric figures should be defined. One of the considerations in this regard is the recognition of hierarchical relationships among quadrilaterals. Another observation from our analysis concerns the relative brevity of the definitions in the "Primary" section of the Merriam-Webster Dictionary, which are often shorter than those in the "Kids Definition" section.

While conciseness can be valuable, our findings suggest that using fewer words does not necessarily enhance clarity or precision. In some cases, essential properties of geometric figures are omitted, leading to conceptual ambiguities. Consider, for instance, the following definition: "An isosceles trapezoid is a trapezoid in which at least one pair of opposite sides are congruent" (Usiskin & Griffin, 2008, p. 42). According to this definition, a parallelogram with non-right angles could be interpreted as a special case of an isosceles trapezoid. However, not all parallelograms are symmetric. Therefore, they do not satisfy the formal properties typically associated with isosceles trapezoids. In such definitions, the use of symmetry in the definition is essential to avoid misclassification and ensure mathematical accuracy. An accurate hierarchical definition would state: "An isosceles trapezoid is a trapezoid that is symmetric about at least one line passing through the midpoints of its opposite sides." Such definitions underscore the importance of ensuring that dictionary definitions, especially those accessible to a broad audience, align with both mathematical rigor and pedagogical clarity.

In this study, we focused on the ways in which definitions of quadrilaterals are presented in the Merriam-Webster Dictionary. However, our study does not investigate how definitions in the Merriam-Webster Dictionary are used within teaching and learning context. Consequently, the present study falls short in terms of the practical use of the dictionary, as it does not reflect on how stakeholders in education interpret dictionary definitions in classroom instruction, tutoring practices, or informal learning settings. In this context, further research is needed to explore how, for instance, students make sense of mathematical definitions encountered in online dictionaries, how these definitions are examined and negotiated within classroom discourse, and how they are aligned with the goals of instruction. Such investigations have the potential not only to inform the refinement of mathematical definitions in digital resources, but also to enhance our understanding of the role these resources may play in shaping teaching and learning processes. As online dictionaries continue to evolve, collaboration with mathematicians and mathematics educators may become increasingly essential to prevent the perpetuation of misconceptions and to support learners at all levels.

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