Scientia in educatione 1(1), 2010
p. 73–85
ISSN 1804-7106

Antropodidaktický přístup k interakcím učitel–žák ve výuce matematiky na 1. stupni školy

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Abstrakt

Článek se zabývá interakcemi jako jednou z hlavních složek učitelova didaktického působení. První část je věnována charakterizaci formy těchto interakcí ve třech různých didaktických kontextech: v „předávaném kontextu“, „intermediálním kontextu“ a „institucionalizujícím kontextu“. Pozornost se soustřeďuje na funkce, které interakce plní v didaktickém systému, a jejich kognitivní efekty. Druhá část se zabývá fatickými interakcemi; jejím cílem je ukázat, jak učitelé postupují, jestliže se snaží sjednotit své vzdělávací poslání se specifickými problémy, které vznikají z různorodosti potřeb jednotlivých žáků.

Klíčová slova: interakce učitel–žák, fatické interakce, matematika na 1. stupni školy.

Anthropo-didactical approach to teacher-pupil interactions in teaching mathematics at elementary school

Abstract

The paper focuses on interactions as one of the main constituents of a teacher's didactical activity. The first part discusses characterization of the form of these interactions in three contrasting didactical contexts: the "devolving context", "intermediary context" and "institutionalizing context". The functions that they fulfill in a didactical system and their cognitive effects on pupils are studied. The second part deals with phatic interactions with the goal of showing how teachers proceed when trying to unite their educational mission with specific problems springing from various needs of different pupils.

Key words: interactions teacher-pupil, phatic interactions, elementary school mathematics.

Interactions have always been regarded as solid ground for exploration of the phenomena of socialisation, school affiliation or for social development of intelligence (especially in Piaget's socio-constructivist perspective). We perceive them as one of the most vital constituents of the teacher's didactical activity, and thus subject to study of conditions of education. It has been known since Piaget that in order to learn, pupils must interact with the object of learning. The Theory of situations (Brousseau, 1998) allows us to model the properties of the knowledge "in play" in a particular milieu in order to enable these interactions. In case of traditional teaching, organisation of the milieu is rarely a-didactical: It is only in very exceptional cases that pupils have the opportunity to return to their decisions that they have taken about the assigned problems. There is no doubt these retroactions are the pre-condition for development of pupils' knowledge. In consequence the teacher "must" teach (in the traditional sense), i.e. he/she must employ various didactical strategies: give examples, say explicitly or suggest through the use of Topaze effect what he/she expects from them (see Novotná, Hošpesová, 2009); these strategies are necessarily more or less compatible both with the not a-didactical nature of the milieu and with his/her pedagogical beliefs. That is why we find it so important and interesting to study interactions teacher–pupil in detail. In these educational contexts, they can be understood as forms of adaptation to this not-adidactivity. This is what we pursue in this text in which we analyse verbal interactions between the teacher and pupils.

The first part focuses on characterisation of the form of these interactions in three contrasting didactical contexts: the "devolving context", "intermediary context" and "institutionalising context". It also shows the functions that they fulfil in a didactical system and finally studies their cognitive effects on pupils. In the second part we focus on a special category of interactions that we named "phatic". Our aim is to show that teachers try to harmonize their teaching goal (to teach a particular notion in their class) with the various problems that arise when working with different pupils individually (good or weak ones).

1 Structure of the interactive field and functions of interactions

1.1 Description of the study context

The research was carried out in 7 elementary school classes (9-year old pupils), $N = 142$ pupils. Each teacher taught two lessons (L_1 and L_2) on solving problems of the type TTT (Vergnaud, 1983); the period between the two lessons was 10 days and the lessons were preceded by a pre-test and followed by a post-test containing 22 problems with only two numbers (smaller than 10).

Three types of contexts were defined:

a) *"Devolving" context.* Teachers regularly use group work without restricting only to this organization form; their lesson is a strongly interactive place; institutionalisation of the model of solving process is largely differentiated in the course of the lesson.

b) *"Institutionalising" context* is characterized by a weak opening and low diversity of situations. Teachers institutionalize the solving procedure very fast; consequently pupils are asked to apply the procedure in various problems; in

short, they try to cope with the set of parameters of the teaching situation as well as possible.

c) *"Intermediary" context* is closer to the institutionalising context, especially in case of poor pupils, even though teachers sometimes try to "open" the situations. However, in contrast to the "institutionalising" context pupils have more opportunities to gain experience with situations of "research" (always not a-didactical).

1.2 Grid for observation of interactions

We only focus on didactical interactions, i.e. those for which:

1. it was possible to clearly identify a link with the object of teaching/learning and

2. the people involved were identifiable unambiguously.

Five modalities of interactions were defined:

- **Spontaneous interventions**, labelled ***SI*** (2 modalities): the pupil intervenes without asking to speak or without any teacher's encouragement; the teacher may react to them and make use of them ($SI+$) or not ($SI-$);
- **Requests for participation**, labelled ***R*** (2 modalities): the pupil asks to speak; he/she may get the permission or not ($R+$ and $R-$);
- **Strict directives for participation**, labelled ***D*** (1 modality): The teacher asks the pupil without the pupil's request to speak.

1.3 Results

1.3.1 Didactical functionality of various forms of interaction

Result 1a: The amount of didactical interactions and the degree of pupils' non-participation regardless of the modality greatly varies across the different classes and teaching styles.

Result 1b: The second lesson (L_2) is in the group of the studied classes and in all 3 teaching styles much less interactive than L_1. The number of interactions initiated by the pupil (SI and R) significantly reduces from L_1 to L_2: (Wilcoxon s: z_{SI} s.; $p < 0.01$ — z_D s.; $p < 0.001$).

This result may be explained by the large amount of time used by the teacher for speaking when concluding the second lesson (necessitated by the institutionalisation!) on the one hand and the decrease of pupil's insecurity and doubts towards the taught subject matter on the other hand. These two things lead to a significant decline of their interventions.

Result 1c: The volume of interactions of all modalities in total is not bound to the level of school achievement of the class — an absence of correlation between the two lessons is obvious; this observation holds for all 3 styles. This phenomenon may be accounted for by the change of teacher's way of posing questions ($\rho = 0.33$; n.s.; $p = 0.34$ for the set of the 7 classes).

The teachers tend to interrogate more often the good pupils in L_1 than in L_2; weak pupils are on the contrary interrogated more frequently in L_2 than in L_1.

What can be observed is a strong correlation between L_1 and L_2 in the frequencies of SI and R observed — $\rho_{SI} = 0.96$; s.; $p < 0.001$ and $\rho_R = 0.89$; s.; $p < 0.006$.

The interactive forms initiated by the pupil — not controlled by the teacher — remain the same from L_1 to L_2; in contrast those initiated by the teacher do not remain the same: the two distributions (D_{L1} and D_{L2}) are not in correlation.

Comments: **In consequence it is necessary to distinguish between two types of interactive modalities:**

1. The first type embodies interactive forms initiated by the teachers who, in consequence, can control them;
2. The second type corresponds to the forms that the teacher can authorise and even prefer without having the chance to control the pupils who take over the initiative in it. For some teachers these interactive forms are didactically functional; it is thanks to these forms that their lessons can proceed:
   - either by functional reduction of the distance between the two types of submission: submission to their pedagogical beliefs on the one pole ("Pupils *must* be active, autonomous") ***and*** submission to a *non a-didactic* organization of the milieu that *in fact* precludes realization of his/her ideals on the other pole. In other words, these milieus do not allow *validation* by retroaction — but only *evaluations* by the teacher. This phenomenon can be clearly observed in case of "Devolving" teachers and in a more graded way in case of "Intermediary" teachers;
   - or, on the contrary, by limitation, reduction of these types of interactions, which is the case of "Intermediary" teachers who regard them as a form of *didactical noise* which one should eliminate as much as possible. However, for the "Devolving" teachers there interventions are didactically vital.

An interactive form does not therefore *bear* a signification and an educational function linked to it. The study of various registers of teachers' deliberateness allows us to understand their mode of didactical action as a product of adaptation to multiple submissions, i.e. to a "wide milieu".

1.3.2 Interactive strategies: Partners of interaction

How can one account for teachers' reasons for "decisions" related to inter-actors?

Result 2a – what could be observed was a strong variability of average scores of solicitations for the pupil's initiative across the different styles on the one hand and among pupils' level of school achievement on the other hand; e.g. good pupils in devolving classes ask to speak 6.35 times (on average) in the 2 lessons while good pupils in institutionalizing classes ask to speak only 3.63 times.

Regardless of their schools level, pupils in institutionalizing classes take much more active part significantly than they are asked to (Wilcoxon: "GOOD" $p. < 0.14$; "AVE" $p. < 0.008$ and "WEAK" $p. < 0.001$), which is in contrast to what can be observed in the devolving and intermediary classes.

Discussion: This result stresses the importance of employment of anthropo-didactical framework for understanding this double variability: inter-pupil (including

intra-style) and inter-style (including the same school level). In fact, if the didactical position of a good pupil (or a weak one) may be defined as a position of the strongest (weakest) probability of satisfying the teacher's expectations, one must say that there remains nothing else but the fact that this participation always depends on objective conditions of its realisation. In other words, "good pupils" and "weak pupils" are to be regarded as prerequisite differentiations necessary for the functioning of the whole didactical system; they correspond to the didactical roles assigned to individuals for the sake of didactical functioning. The importance of these roles is in the search for the function that they fulfil, not in the personal characteristics (social, psychological or others) of the actors (for the last aspect see the works of anthropologists of education Mc Dermott, 1977; Gumperz, 1989; Gearing, 1973).

Finally, the inter-style variability that could be observed clearly shows that the interactions linked with the same role may strongly vary from one context to another: "good pupils" assume the responsibilities (obligations, ways of existence...) connected to these non-equivalent roles with respect to their didactical function.

Result 2b – Utterances from the pupil's initiative (SI) and teacher's interrogations (R) are not interrelated ($\rho = 0.17$; n.s.) in the 2 lessons L_1 and L_2 in the studied group. This result is the same for each of the 3 styles ("Devolving": $\rho = 0.23$; n.s.; "Intermediary": $\rho = 0.28$; n.s.; "Institutionalising": $\rho = 0.15$; n.s.).

Thus, the reason why pupils do not ask to speak is not necessarily that they are explicitly called by the teacher. In case of all 3 styles, the weakest pupils who are least often called by the teachers and simultaneously they are the group who ask to speak least often.

1.3.3 Cognitive effects of interactions teacher–pupil

The relevant literature suggests that interactions with teachers support pupils' progress in their school education and this idea is accepted as a given fact. However, our research shows that this presumption does not hold:

Result 3a – The sample shows no correlation between the amount of effective interactions ($SI+$, $R+$ and D) and the achievement in the post-test ($\rho = -0.04$; n.s.; $p = 0.64$). The result remains the same at each school level and for each of the three school styles.

Results 3b – Interactions initiated by the teacher (D) are not without cognitive effects on the pupils; these effects are to be observed in good pupils from "institutionalising" and "devolving" classes:

- In "devolving" classes, the more the good students are prompted by the teacher, the less progress they show in the post-test ($\rho = -0.535$; $p < 0.06$);
- Inversely, in the "institutionalising" classes, the more the good students are prompted by the teacher, the more progress they show in the post-test ($\rho = -0.551$; $p < 0.08$).

1.4 Conclusion to the first part

Contrary to what is generally accepted about cognitive effects of verbal interactions, these analyses clearly show that it is neither the pupils, nor the most interactive classes who show the biggest progress in the post-test. Moreover, the same interactive mode had by no means the same didactical effect in different didactical contexts:

It could for example be observed that the teacher's interrogations (D) have radically different effects on good pupils ("Devolving"/"Institutionalising").

These results could appear contradictory if we did not consider in their interpretation the didactical function these interactive forms may assume. "To interrogate a good pupil" does not have the same function for "Devolving" and "Institutionalising" teachers: Interrogation of the best pupils is in case of "Devolving" teachers didactically functional in case that it enables them, thanks to a change of perspective, proceed in the lesson without overtly disclosing their intent to teach. Let us remark at this point that this behaviour opposes their pedagogical intent. In contrast, the same activity would in case of "Institutionalising" teachers result in an intended change of pupils' relation to the subject matter. In other words: "To interrogate a good pupil" represents to a "Devolving teacher" a task oriented on teaching and therefore its function is regulation of the teaching process, while in case of "Institutionalising" teacher it aims at the pupil and its declared (but not necessarily actual) function is regulation of the pupils' learning.

2 Anthropo-didactical approach to one type of interactions: phatic interactions

The following part presents a research on the effects of one category of interactions called "phatic" (e.g. expressions such as "we will see later"). Its aim is to make a report on how teachers solve the classic equation of efficiency of teaching on the one hand (to allow the highest possible number of pupils to learn) and of its equity on the other hand (to allow each pupil to profit from teaching/learning). The research allows exploration of the question of individualisation of teaching/learning. In fact, individualisation of teaching/learning has become a mode privileged and even recommended by ministerial directives (in France) for realisation of efficient and equitable education.

2.1 Theoretical framework and questions

This research is registered in a programme known as anthropo-didactical (Sarrazy, 2002a, 2007; Marchive, 2006). Its fundamental idea can be formulated as follows: to understand the teacher's work means to examine the effects of various systems of submissions (anthropological and didactical) to which it is subordinated.

2.1.1 Didactical and anthropological limitations to the teacher's activity

Three systems are considered here:

1. *Didactical submission:* the teacher must teach curricular subject matter (by which we do not mean a particular discipline but a particular topic of a lesson for which s(he) has to connect a certain number of conditions — milieu in Brousseau's sense, 1998);

2. *Institutional submission:* the teachers are asked e.g. to individualise teaching in order to increase the efficiency of their activity and to modify the content of the knowledge to the demands of the public (for synthesis, see Roiné, 2009);

3. *Pedagogical* (or ethical) *submission* through which the teachers try to make the greatest possible number of pupils successful and to avoid leaving too many of them neglected.

The three imperatives often appear (even in the heart of the teaching action) as contradictory. What was noted in course of the analysed interaction was that "participation of a good pupil can be an obstacle in whole class discussion; at the beginning of the lesson the teacher often asks good pupils to be silent because they disclose too early what is to be discovered, there is no longer any suspense for the rest of the class and with no suspense, all interest in the game is lost; at the same time participation of a weak pupil makes the whole class discussion difficult, it confuses the paths and diverges the teacher into dangerous and too distant waters; in case of average pupils the teacher can control the course of the lesson more easily, s(he) gently distils these contributions through a subtle game in which s(he) poses his/her own way the questions — answers — resettings and through which s(he) manages to follow his/her course. This abides to the strict law of didactical economy: allow the greatest possible number of pupils to gain knowledge in a limited period of time. Many young teachers have a lot of difficulties to combine all these requirements in their teaching practice. Be it for ethical or pedagogical concern, they engage in endless interactions, often of argumentative nature with weak pupils, they get lost in the numerous and tortuous meanders of their lacunas and together with them the rest of the class." (Sarrazy, 2001a).

The presented research has the following aim: to understand how experienced teachers (all of them have at least 5 years of professional practice) solve the above mentioned tripartite equation (didactical, institutional and pedagogical).

2.2 Phatic interactions

In order to study this issue we decided to focus on one special type of interactions between transmitter and receiver of the teaching/learning relationship. In the framework of functions of communication of Jakobson (1963), we will study interactions called "phatic". These interactions are often unstable. What proved to be an especially useful tool for identification of these interactions were video recordings. Basically, the function of phatic interactions is the strive to keep the communication canal between the teacher and the pupil. In fact, these interactions allow pupils to stay in the interactive field in the course of the lesson. Phatic interaction by no means forms a tool for handling the teacher's answer, be it cognitively (no explicit validation or explanation) or didactically (the teacher does not use it to demonstrate pupils what he/she wants to do or not to do).

This type of interaction is a good instrument for regulation of didactical time. We have already shown in our work (Chopin, 2007) that the interactive assortment of interactions used by the teacher in order to progress in the lesson has its conspicuously temporal logic in the didactical meaning of the term. This logic of didactical time, time of construction of new knowledge in the classroom, is relatively autonomous (but not entirely independent) of the teacher's pedagogical background and of the effective time available for realisation of teaching (*Ibid.*). For example: we have already shown that ostensibly contrasting interactive forms (e.g. closed and directed by the teacher's strict limitation of the interactive field, or open and flexible when the teachers leaves as much space for spontaneous interactions as possible, asks pupils to justify their answers, etc.) often fulfill similar functions in the

progress of didactical time depending on how they are used (e.g., publicly, in front of the whole class, or in a more individualized way or only for some pupils).

To put it briefly, the presented study about phatic interactions in teaching belongs to the set of works simultaneously focusing on interactions in their full didactical dimension (Sarrazy, 2001b) and on the modes of regulation of heterogeneities in the construction of didactical time (Sarrazy, 2002b; Chopin, 2007, 2010),

2.3 Experimental setting

8 classes (197 10-year-old pupils) participated in the study. We video recorded 24 one-hour meetings on solving problems called "TTT"[1]. These problems correspond to the fourth additive structure in Vergnaud's (1990, 1994) typology. This problem type may be illustrated by the following typical problem:

Lou plays two games of marbles.
She plays one game. In the second game she loses 4 marbles.
After two games she possesses 6 marbles.
What happened in the first game?

This type of problems was presented to each teacher (individually) without giving them any special signals on how to organize their teaching. The only objective was to allow their pupils to improve results from the pre-test (teachers did not know these results). The teachers also agreed to teach a lesson on this topic.

Pupils solved:

- A pre-test consisting of 22 TTT problems of various difficulty;
- A post-test, after the set of the lessons (identical with the pre-test, it allows measuring of pupils' progress using an index of progress Ip).

The pre-test indicated that the 197 pupils could be divided into three levels ("good", "average", "weak"). It is of interest to note here that the classes did not differ in the perspective of the school level ($\chi 2 = 15.18$; $p = 0.37$).

For a more detailed study of interactions the video recorded teaching sessions were watched for several times and consequently a grid for analysis of the interactive field in the set of lessons was created. In the analysis we focused only on the category of phatic interaction.

Phatic interactions are investigated in the framework of those types of interaction where the teacher must deal with the pupil's intervention in the lesson. We may consider the following two situations:

- The teacher posed a question and a pupil suggests an answer that does not correspond to the teacher's expectations;
- A pupil intervenes spontaneously in the course of the lesson suggesting an answer, a way of solving, etc.

In the two situations the teacher faces the following alternative:

[1] The particularity of this structure is that it involves only positive or negative transformations, there is no indication of the initial numeric stage — hence the name "TTT" (1st Transformation — 2nd Transformation — Transformation composite).

- He/she may come with cognitive or didactical treatment of the pupil's intervention (either pursuing an interactive exchange according to regulations, or validating or invalidating the pupil's proposal giving the pupil a didactical status useful for the rest of the class);

- He/she may come with phatic treatment of the intervention, i.e. adopt it without taking into account cognitive regulations or didactical treatment — phatic treatment includes teacher's reactions such as "Yes, yes, this is very interesting but we will see it later", "maybe ... any other ideas?", etc.

2.4 Results and analyses

Phatic utterances in teacher–pupils interactions can also be calculated proportionally to the total amount of interactions in each class. These proportions may then be linked to the relative data about the pupils' progress.

2.4.1 A positive relationship between the pupils' progress and the proportion of phatic interactions

The results of our analyses show a highly significant relationship between the pupils' progress and the amount of phatic interactions. In other words, the higher the proportion of phatic interactions is (with respect to other types of interactions by the same teacher) the more the average level of pupils grows ($r = 0.84$; s.; $p. < 0.01$). This is illustrated by the figure 1.

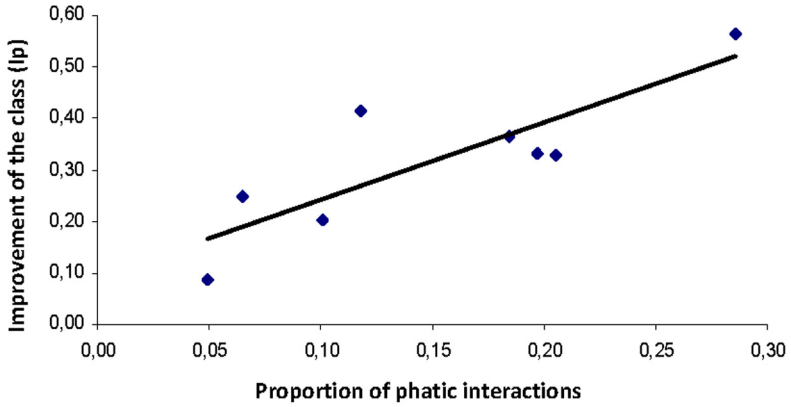

Figure 1: Correlation between the proportion of phatic interactions and pupils' improvement

This result must appear more than surprising. How can an absence of cognitive or didactical treatment of pupils' interventions in the lesson support their improvement? This can be explained in the anthropo-didactic framework presented above. We interpret it as a manifestation of time pressure imposed on the teacher. If phatic interactions do not just merely mechanically push ahead the didactical time, they allow the teacher to guarantee an "optimal" balance between didactical time (pupils' progress) and legal time (clock time). In fact if the interaction is to be an efficient tool for the progress of didactical time, the teacher must guarantee of their distribution and their content. Consequently, treatment of interactions in a phatic mode that he/she does not perceive as useful for development of knowledge of the greatest number of pupils becomes an efficient and equitable tool for progress of

the whole class. This interpretation is confirmed by the fact that the correlation between the proportion of phatic interaction and pupils' progress is notable in case of average pupils ($r = 0.87$; $p. < 0.01$), more ambiguous in case of weak pupils ($r = 0.57$; $p. < 0.10$) and totally insignificant in case of good pupils ($r = 0.02$), all this independently on the total amount of interactions. The phatic component of teacher-pupil interactions thus seems to be the tool for promoting progress of "the big crowd" without helping the weakest pupils (or the best pupils). Therefore the role played by phatic interactions in the interactive mode of teaching is ultimate. What remains is to specify the incentives leading to this kind of interactions. The following results make it possible to explore the issue in several perspectives.

2.4.2 Role of phatic interactions in the teaching process

If we are to understand the role the phatic interactions play in the teaching process it is necessary to examine in more detail how they are used by the teacher. We were especially interested in the nature of their distribution in the interactive game of the class, depending on the pupils' school level.

Table 1 below summarises the data necessary for examination of the relationship between:

- the progress of pupils in each class (indicator Ip, the second column in the table) — the higher the indicator, the greater the progress of the class;
- the significance level in Chi-squared test (the third column in the table) that enables to examine if the distribution of phatic interactions in each class is significantly differentiated according to the pupils' school performance — the weaker the level of significance, the more the phatic interactions are distributed with respect to the school level of pupils to whom they are addressed (i.e. the teacher does not use the phatic interactions in the same way in case of good, average and weak pupils).

The last three columns in the table show the attractions and repulsions (the sense of deflection of independency). They allow to state what kind of pupils is addressed by the teacher's phatic interactions (good, average and weak):

Table 1: Progress of classes and distribution of phatic interactions

Class	Progress (Ip)	Significance level of Chi-squared	Attractions/repulsions of phatic interactions		
			Good pupils	*Average pupils*	*Weak pupils*
Eco1	0.33	0.21	+	−	+
Eco2	0.56	0.00	−	−	+
Eco3	0.41	0.01	−	+	−
Eco4	0.33	0.05	−	−	+
Eco5	0.09	0.52	−	+	+
Eco6	0.20	0.97	−	−	+
Eco7	0.25	0.69	+	+	−
Eco8	0.36	0.03	−	+	−

As it can be seen above, the classes that progress most are those where the distribution of phatic interactions is significantly differentiated according to the school

level. Analogically, the classes where the distribution of phatic interactions is not differentiated show the weakest progress ($r = -0.73$; s.; $p < 0.05$). A more qualitative analysis of the last three columns shows that if there exists a differentiated use of phatic interactions by the teacher in dependence on his/her pupils' school performance (level of significance of Chi-squared smaller than 0.05), the average and weak pupils are the main addressees of phatic interactions. This confirms the hypothesis that the teacher manages through this interactive game simultaneously to keep alive the canal of communication with his/her pupils without slowing the didactical time for the whole class.

2.5 Conclusion

Individualisation of education by adaptation to (namely cognitive) characteristics of individuals is often presented as a privileged strategy of schools capable to face heterogeneity of their public and to reduce pupils' difficulties. Ministerial directives addressed to teachers clearly reflect this policy. The following quote cites the recent programmes for French primary education in 2008:

> "[...] the pupils with difficulties have to be able to benefit from the personalised and differentiated help since the first difficulties occur and before they are not irreversibly installed. [...] The role of the teacher is [...] to help his/her pupils to progress in mastering the objectives fixed by the programmes [...] he/she should choose the methods mostly adapted to individual characteristics and specific needs of his/her pupils." (M.E.N., PROG, 2008, 10–11).

Undoubtedly many teachers try to follow the idea of this ambitious project to make each pupil progress, and especially to help pupils "with difficulties". Despite that in practice they will be accused of not leaving enough space for this individualisation. We firmly believe that it is crucially important to understand the reasons for this accusation, even if it means that the explanation of the cause of existence of the hiatus should be restricted to inertia of the educational system (or even to conservatism of the teachers). It was one of the aims of this study.

What is clear from the here presented research, teacher-student interactions are undoubtedly a privileged tool of individualisation in teaching. The teachers profit from verbal exchanges with their pupils. It helps them uncover their difficulties and, if possible, to rectify them. In order to meet this requirement some teachers, especially the youngest and least experienced, engage in long and costly interactive exchanges with their pupils, accommodating requests for explanations, repetitions and reformulations, also attempting to treat errors of pupils (namely of the weak ones) as they appear. Our results clearly show the limits of such a practice.

Regardless of pupils' school performance, the pupils' progress is triggered by the use of phatic interactions, i.e. interactions that eliminate or modify cognitive treatment of pupils' propositions. More precisely, these progresses are the most important because these interactions specifically address the average and the weak pupils (i.e. those who are considered to be the target group profiting from individualised teaching).

The anthropo-didactic perspective helps to understand this phenomenon as the result both of submission to didactical time (advance the knowledge for the highest possible number of pupils in the given time) and of pedagogical (or ethical) submission (not to neglect pupils). In this framework, the function of phatic interaction

becomes essential. Without it, the teacher loses all flexibility in the management of his/her teaching. Too long an exchange with a pupil endangers the progress of the lesson for the rest of the class. In other words, conditions of efficiency and equity of teaching are to be looked for in existence of a favourable proportion of phatic utterances, i.e. utterances that may be *a priori* perceived as void of all didactical quality and which are despite that absolutely fundamental for harmonization of these two imperatives that are unfortunately far too often perceived as contradictory.

Bibliography

BROUSSEAU, G. *Théorie des situations didactiques* [textes rassemblés et préparés par N. Balacheff, M. Cooper, R. Sutherland, V. Warfield], Grenoble, La Pensée Sauvage, 1998.

CHOPIN, M.-P. *Le temps didactique dans l'enseignement des mathématiques. Approche des modes de régulation des hétérogénéités didactiques.* Thèse pour le doctorat de l'Université Victor Segalen Bordeaux 2, 2007, 337 p.

CHOPIN, M.-P. Les hétérogénéités: quels critères, quelles fonctions?, In R. Rouchier et al., *Actes de la XIIIème école d'été de Didactique des Mathématiques* (Sainte-Livrade, Lot-et-Garonne, du 18 au 26 août 2005) , Cédérom d'accompagnement. Grenoble: la Pensée Sauvage, 2008.

CHOPIN, M.-P. Le temps didactique et ses niveaux d'étude: enjeux d'une clarification conceptuelle pour l'analyse des pratiques d'enseignement, *Recherches en didactique des mathématiques*, 30(1), 2010, p. 83–112.

JAKOBSON, R. *Essais de linguistique générale. 1. Les fondations du langage.* Paris : Minuit, 1963.

MARCHIVE, A. *Approche anthropo-didactique des phénomènes d'enseignement et de formation: contribution à l'étude des rapports entre pédagogie et enseignement*, Note de synthèse pour l'habilitation à diriger des recherches, Université de Bordeaux 2, 2006.

Ministère de l'Éducation Nationale. *Préambule des programmes de l'école primaire. (Arrêté du 9 juin 2008).* Bulletin officiel hors-série no. 3 du 19 juin 2008.

NOVOTNÁ, J.; HOŠPESOVÁ, A. Effet Topaze et liaisons dans les pratiques des professeurs de mathématiques. In *Espace mathématique francophone.* Dakar, Sénégale, in print, 2009.

ROINÉ, C. *Cécité didactique et discours noosphériens dans les pratiques d'enseignement en S.E.G.P.A.: une contribution à la question des inégalités.* Thèse pour le doctorat de l'Université Bordeaux 2, 2009, 404 p.

SARRAZY, B. *La sensibilité au contrat didactique*: Rôle des Arrière-plans dans la résolution de problèmes d'arithmétique au cycle trois, Thèse pour le doctorat de l'Université de Bordeaux 2 — Mention Sciences de l'Education, sous la direction de M. le Professeur Pierre CLANCHÉ, 1996, 775 p.

SARRAZY, B. Les bulletins scolaires ne servent-ils qu'à évaluer les compétences des élèves ? Contribution à l'analyse des fonctions didactique et pédagogique des appréciations, *Les sciences de l'Education pour l'Ere nouvelle*, Vol. 33, No. 3, 2001a, 51–81.

SARRAZY, B. Les interactions maître-élèves dans l'enseignement des mathématiques: Contribution à une approche anthropo-didactique des phénomènes d'enseignement, *Revue Française de Pédagogie*, 136, 2001b, p. 117–132.

SARRAZY, B. Les hétérogénéités dans l'enseignement des mathématiques. *Educational Studies in Mathematics*, 49, 2002, 89–117.

SARRAZY, B. *Approche anthropo-didactique des phénomènes d'enseignement des mathématiques: fondements épistémologiques et ancrages théoriques*, dans G. Gueudet, Y. Matheron (éds.). *Actes du séminaire national de didactique des mathématiques – Année 2006*, Paris : IREM Paris 7, 2007, pp. 79–99.

VERGNAUD, G. *L'enfant, la mathématique et la réalité:* Problèmes de l'enseignement des mathématiques à l'école élémentaire, Berne : Peter Lang, 1983, 217 p.

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